



Inquiry-Based Learning in Science: Mathematical Reasoning's Support of Critical Thinking

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Abstract

The evolution of educational paradigms toward inquiry-based learning (IBL) in science education has significantly altered pedagogical practices by emphasizing the necessity of active student participation in the learning process. Through questioning, research, and problem-solving, IBL invites students to investigate scientific phenomena, so promoting a closer knowledge retention and understanding. Integral to this approach is the function of mathematical thinking, which functions not only as a tool for quantitative analysis but also as a basic framework for critical thinking. Mathematical reasoning helps students organize their searches, efficiently analyze data, and come to reasonable conclusions. This interaction improves their ability to link mathematical ideas and scientific concepts, developing more advanced higher-order thinking abilities. Understanding how mathematical reasoning supports critical thinking within IBL will help curriculum development and teaching strategies in science education be much more informed as education progressively prioritizes integrating multidisciplinary approaches. In science education especially, IBL is especially helpful since it fits very nicely with the Next Generation Science Standards, which support student-centered learning environments that advance critical thinking and teamwork. IBL helps students develop critical skills, including analytical thinking and reasoning, by pushing them to create their own questions and search for answers through investigation. Moreover, including mathematical reasoning in IBL improves students' problem-solving capacity by letting them approach challenging scientific questions with a strong methodological framework. IBL thus not only fosters curiosity but also provides the cognitive tools required for advanced learning in scientific fields.

Keywords: Critical Thinking, Inquiry-Based Learning, Mathematical Reasoning, Student-Centered Learning

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1. INTRODUCTION

The shift in educational paradigms toward inquiry-based learning (IBL) in science education has significantly altered pedagogical practices by highlighting the importance of active student participation in the learning process. IBL encourages students to look into scientific events through inquiry, research, and problem-solving, which improves knowledge retention and comprehension. Essential to this approach is the function of mathematical reasoning, which forms a basic framework for critical thinking and a quantitative study tool. Mathematical thinking helps students organize their questions, properly evaluate data, and come to reasonable conclusions. This interaction improves their ability to link mathematical ideas and scientific ideas, thus developing their higher-order thinking ability. Understanding how mathematical thinking supports critical thinking inside IBL will greatly guide curriculum development and teaching strategies in science education, as education prioritizes integrating multidisciplinary approaches.

Emphasizing the active participation of students in the learning process via inquiry, questioning, and problem-solving, IBL is an instructional strategy. Using hands-on experiences and reflective thinking, this pedagogical framework invites students to interact with materials, promoting closer knowledge retention and comprehension. In science education especially, IBL is helpful since it fits very nicely with the Next



Generation Science Standards (2015), which support student-centered learning opportunities that foster critical thinking and teamwork. IBL helps students develop critical abilities, including analytical thinking and reasoning, by motivating them to create their own questions and search for answers through research. Moreover, including mathematical thinking in IBL improves students' capacity for problem-solving so that they may approach challenging scientific questions using a strong methodological framework. IBL thus, only stimulates interest and gives pupils the cognitive skills required for advanced study in scientific fields.

Mathematical thinking improves understanding and problem-solving capacity. Hence, it is valuable in many different fields, including science. Reasoning is fundamental in mathematics since it entails both simple heuristics and sophisticated cognitive processes, which helps one to understand complicated relationships and enhance educational results (Morsanyi et al., 2018). Mathematical thinking is crucial for completing assignments in physics education; creative thinking is especially crucial for understanding courses and obtaining better grades (Johansson, 2016). Beyond simple formula manipulation, this thinking promotes a deeper conceptual understanding of physical principles and emphasizes the interdependence between mathematics and physics (Uhden et al., 2012). Integrating quantitative thinking into courses has been shown in biology to help students apply mathematical skills to biological settings, proving that mathematical reasoning can improve learning without compromising biological ideas (Hester et al., 2014). Moreover, historical studies of eminent scientists such as James Clerk Maxwell and Michael Faraday show the cognitive function of mathematics as a means of representation, underlining its significance in scientific thinking and invention (Tweney, 2009). These ideas, taken together, highlight how important mathematical thinking is for computation and basic scientific research and education across many fields.

A fundamental pillar of scientific study, mathematical reasoning develops critical thinking abilities necessary for efficient evidence-based decision-making and problem-solving. Through interacting with mathematical ideas, students improve not only their understanding of scientific events but also their capacity to build rational arguments and assess the validity of their results. In research-based learning settings, when students are urged to hypothesize, gather data, and evaluate results using quantitative measures, this synergy between mathematics and science is especially evident. This approach requires a strong awareness of mathematical ideas and scientific reasoning, so underlining the need of teacher professional development to stress these related fields (Jimenez-Aleixandre et al., 2010). Thus, arming teachers with the skills to include mathematical ideas into science courses will improve teaching strategies and student learning in scientific literacy (van der Molen et al., 2010). This whole integration emphasizes that mathematics is important in developing a strict scientific attitude.

Especially in the sciences, the link between IBL and critical thinking is becoming more and more important for advancing educational approaches. By actively pushing students to interact with scientific ideas through exploration and problem-solving, IBL helps them develop critical thinking and positions them as co-creators of knowledge rather than consumers. A pedagogical method called IBL greatly improves critical thinking abilities in many different kinds of classrooms. By actively asking students to interact with materials and hone their problem-solving skills, IBL promotes critical thinking by encouraging students to investigate, challenge, and build their knowledge (Sam, 2024). By encouraging intellectual activities that challenge students to interact with music outside of technical proficiency and fostering a balanced development of musicality and independent musicianship, IBL has been demonstrated in music education to improve critical thinking (Costs-Onishi & Kwek, 2022). The context in which IBL is used and students' preparedness to participate in self-directed learning will, however, determine how successful IBL is in fostering critical thinking (Tang, 2020). The context-dependent character of critical thinking implies that although IBL can be a useful tool, its success varies across disciplines, such music, art, and mathematics, each requiring customized approaches to teach and evaluate critical thinking skills (Rickert, 1967). Moreover, using IBL in higher education has shown favorable student reactions, suggesting its possibility to improve educational results in many spheres, including philosophy, journalism, and engineering (Friedman et al., 2010).

This pedagogical change is basic since research deepens knowledge and encourages the application of sophisticated analytical skills and reasoning needed for scientific inquiry. As the literature notes, for example, good learning environments that apply inquiry ideas start higher-order thinking fundamental in building knowledge frameworks (Jimenez-Aleixandre et al., 2010). Furthermore, in line with discovery-based learning models, inquiry-supported thinking lets students negotiate several content areas, thus improving their capacity to critically assess evidence and express logical arguments (Abrahamson, 2015).

IBL is therefore a very important tool for developing the critical thinking abilities required for scientific literacy.

Notwithstanding these advantages, issues including the necessity of thorough teacher training and supportive learning environments have to be resolved to maximize the effect of IBL on the growth of critical thinking (Sam, 2024). All things considered, IBL offers a good path for developing critical thinking as long as its application is closely matched to the particular learning environment and supported by sufficient tools and instruction.

2. THEORETICAL FRAMEWORK OF INQUIRY-BASED LEARNING

Emphasizing active participation, critical thinking, and the construction of knowledge by inquiry and questioning, the theoretical framework of IBL is a student-centered pedagogical approach. IBL develops critical thinking abilities necessary for scientific research as it motivates students to ask questions, create hypotheses, and do investigations. Many design heuristics that support learning, such productive failure and problem-posing, enhance this participation (Abrahamson, 2015). These approaches help students to link mathematical ideas to scientific concepts and improve their capacity to negotiate challenging problems. Mathematical reasoning is still absolutely essential even if some questions about the effectiveness of discovery-based methods—especially the difficulty of moving from naive ideas to historically established concepts—particularly call for attention. It helps students to develop their cognitive skills and deepen their knowledge of scientific approaches, so strengthening their educational experience in the sciences and contributing to their resilience.

As seen by its application in several settings, including preschool science education, socio-scientific research, and geology (Muñoz, 2023; Ariza et al., 2021), this pedagogical framework is flexible across many educational levels and disciplines. IBL helps young learners to participate in scientific inquiry by play and exploration, so fostering a culture of inquiry (Ramanathan et al., 2021). It also changes teachers in preschool environments from knowledge transmitters to facilitators. In socio-scientific settings, IBL is combined with education for environmental citizenship, so fostering transdisciplinary knowledge and socio-political action—both of which are absolutely vital for tackling difficult environmental problems (Ariza et al., 2021). The framework also spans online environments, where scaffolding methods including the POEE model (Predict, Observe, Explain, Evaluate) support self-regulation and active exploration, so improving the inquiry process using digital tools (Mamun, 2022). In geology, IBL entails students in hypothesis development, data interpretation, and group discussions, so fostering scientific skills that conventional techniques may impede (Muñoz, 2023). To solve issues including instructional support and student readiness for self-directed learning (Sam, 2024), IBL must be effectively implemented with thorough teacher training, supporting learning environments, and technological integration. All things considered, the IBL framework is flexible and successful in improving educational results by encouraging critical thinking, creativity, and academic excellence over many educational environments.

Constructivist theories emphasizing active participation in the learning process—that is, in contrast to conventional rote memorization methods—have shaped IBL historically. Emphasizing student-centered learning and critical thinking, IBL has developed as a major pedagogical tool across many fields. Early educational thinkers like John Dewey argued for experiential learning to develop critical thinking and underlined the need for student research to advance a better knowledge of scientific ideas. For example, a study found that students involved in a community of inquiry showed increased competency in Science and Mathematics by means of cooperative design activities, so clearly departing from passive learning settings (Nichols et al., 2022). This evolution points to a growing awareness of IBL as a transforming teaching tool that improves mathematical reasoning and develops the critical thinking abilities necessary in negotiating difficult scientific problems (Busari, 2024).

According to modern studies, including IBL into science courses—especially in mathematics—has clearly shown great pedagogical benefits. IBL has long been used in a variety of educational environments, including the laboratory-based physics course Thacker describes—which has been taught for more than two decades. Without conventional lectures, this course emphasizes students creating quantitative and qualitative models by means of experiments, so greatly improving their conceptual understanding and thinking ability over conventional approaches (Thacker, 2023). While underlining the need of thorough

teacher training and supportive learning environments for efficient implementation, a systematic review shows the good effect of IBL on students's critical thinking, motivation, and academic performance across disciplines (Sam, 2024). In the context of geology, Muñoz shows IBL by means of a case study on sliding rocks whereby students generate hypotheses and interpret data, so developing scientific skills that conventional methods would impede (Muñoz, 2023). Though challenges still exist in moving from campus-based learning to school placements, IBL improves pre-service teachers' knowledge of science concepts and teaching competency in science teacher education (Strat et al., 2023). Costes-Onishi and Kwek point out in music education a dearth of research projects fostering critical thinking; they propose that IBL might improve critical musicality and independent musicianship (Costs-Onishi & Kwek, 2022). These studies underline the difficulties and best practices required for IBL's effective integration into educational curricula and its part in fostering critical thinking and active participation.

Emphasizing student agency, critical thinking, and group inquiry, IBL has a number of basic ideas. Fundamentally, IBL is asking questions that students find relevant, so encouraging natural drive and involvement in the course of instruction. Through hands-on activities whereby they create hypotheses, run experiments, and analyze data, students are urged to actively construct knowledge actively, so strengthening their mathematical reasoning abilities as they understand scientific events. Furthermore, the function of the teacher changes from a conventional knowledge transmitter to a facilitator guiding students in their search paths and encouraging peer conversation. This change is especially noticeable in science education, where students learn to express arguments and reason mathematically, preparing them for challenges in the real world (Jimenez-Aleixandre et al., 2010).

Emphasizing critical thinking, creativity, and active participation, IBL—a student-centered pedagogical method—allows students to investigate, challenge, and build their knowledge of subjects—so enabling them to Its emphasis on critical thinking, problem-solving, and questioning—which transforms the learning process from teacher-led to student-centered—shifts the focus from Wu et al., 2015 from Practiced in many educational levels and disciplines, including science and math, IBL is successful in both undergraduate and graduate environments (Aditomo et al., 2013). Bruder & Prescott, 2013 The method has improved students' academic performance, critical thinking, and drive. Effective implementation of comprehensive teacher training, supportive learning environments, and technological integration is therefore dependent on the need for significant instructional support and varying degrees of student readiness for self-directed learning (Sam, 2024). Challenges include these. By enabling access to pertinent information and enabling teachers to track learning development, developing IBL support systems—such as those including advanced knowledge exploration modules—can help students to achieve better learning results and efficiency (Wu et al., 2015). IBL is generally acknowledged for its ability to raise teaching quality and educational results despite some discrepancies in empirical data (Bruder & Prescott, 2013). The success of IBL depends on overcoming these obstacles and using its advantages to create a more interesting and efficient classroom. Finally, these qualities of IBL improve content retention and foster a stronger capacity to think critically, a necessary ability in the modern educational scene (Stylianidou et al., 2018).

By designing settings that support critical thinking, creativity, and active participation among their students, teachers significantly help IBL to flourish. Their help in enabling IBL is transforming since they lead students through investigative procedures and model mathematical reasoning—a necessary ability to develop critical thinking. Emphasizing the need of identifying patterns and participating in reflective conversation about their observations, educators must set an environment where students feel empowered to ask questions and investigate concepts deeply using approaches like "Noticing and Wondering." As a systematic review of IBL's effect on educational outcomes emphasizes, effective implementation of IBL calls for thorough teacher training, supportive learning environments, and technology integration to help inquiry-based activities to be facilitated. By guiding students through the research process and motivating them to build their knowledge, educators are seen as facilitators who encourage transformation—akin to alchemists—by means of Supported by grants and institutional backing, faculty members from many fields have effectively used IBL in higher education, so highlighting the value of institutional support in enabling IBL (Friedman et al., 2010). In particular fields like physical chemistry, teachers guide classroom conversations whereby students link mathematical and graphical representations to macroscopic and submicroscopic events, so improving their reasoning abilities (Becker et al., 2015). Teachers' sustained professional development greatly enhances the quality of guided inquiry-based instruction, which results in a notable academic increase in students—especially in middle school science classrooms—particularly in terms of These results highlight especially how important teachers are in maximizing the advantages of IBL using strategic facilitation and ongoing professional development, so improving student learning and involvement in different educational environments.

Moreover, by using design research techniques, teachers can modify their teaching plans to better grasp and analyze student actions in the framework of inquiry-based assignments, thus improving the whole learning process in many classrooms (Abrahamson, 2015). Using IBL in science, teachers can combine culturally relevant practices according to a systemic perspective, ensuring that every student from all backgrounds can access fair chances to develop their mathematical reasoning and critical thinking skills.

3. MATHEMATICAL REASONING IN SCIENCE EDUCATION

Fostering critical thinking especially in inquiry-based learning environments depends on including mathematical ideas into scientific education. By means of problem-solving and analysis, this pedagogical approach motivates students to interact with scientific ideas using mathematical tools to interpret data and generate conclusions. Studies show that guided inquiry worksheets and audience response systems improve student participation and deep cognitive processing, so increasing student engagement (Koretsky et al., 2018). Furthermore, analyzing rhetorical theories of argumentation in curricula shows a clear need for methodical integration of mathematical ideas in teaching strategies, usually lacking in many different educational environments (Jimenez-Aleixandre et al., 2010). By including mathematical thinking into scientific research, teachers help students develop their analytical abilities and equip them to create cogent arguments grounded on quantitative data, so improving their scientific literacy. Therefore, the development of competent critical thinkers ready for real-world challenges depends on the interaction between mathematical reasoning and science education.

Inquiry-based learning in science depends on mathematical reasoning, which shows up in several forms that improve critical knowledge and application. One common form is quantitative thinking, which helps students to model scientific events, spot trends, and understand numerical data. When students interact with data sets during experiments, this kind of thinking is absolutely essential to help them to grasp statistical relevance and variability. Moreover, conceptual thinking helps students to relate scientific ideas with mathematical ideas, so strengthening their understanding of fundamental theories (Koretsky et al., 2018). Moreover, metacognitive thinking helps students to consider their approaches of solving problems, so improving self-regulation and scientific inquiry effectiveness. These entwined forms of thinking enable a comprehensive approach to education, arming students with necessary tools for data interpretation and theory application, so bridging the distance between mathematical ability and scientific inquiry (Jimenez-Aleixandre et al., 2010).

Including mathematical ideas into scientific research is a multifarious strategy that improves knowledge and application in learning environments. Many studies clearly show the synergy between mathematics and science, including combining mathematical modeling with scientific investigation to teach ideas like density, which has shown to be effective when using the BSCS 5E instructional model (Manunure & Leung, 2024). < Applying the Fundamental Theorem of Calculus (FTC) through scientific reasoning helps students to develop their covariational reasoning, a vital ability for understanding calculus (Sokolowski, 2021). This integration is also shown in this way. Rich tasks combining mathematics and science are promoted by the Authentic Integration of the Mathematics and Science model, facilitating enhanced learning through inquiry and dialogue and improving problem-solving skills and the application of mathematics to other fields (Treacy, 2021). Furthermore, as shown in motion modeling in a physics context (Sokolowski, 2021), the application of mathematical reasoning-including limits-in physics education has improved students' scientific inquiry skills and general STEM disposition. Moreover, including biology and mathematics in inquiry-based projects helps students to apply mathematical ideas to support scientific research, thus improving their knowledge of scientific events and the accuracy demanded in scientific descriptions (Bakke et al., 2013). This research highlights the need to include mathematical ideas in the scientific investigation to support better knowledge and implementation of STEM education.

Mathematical ideas should be included in scientific research since they enable students to apply quantitative thinking when tackling challenging problems, thus promoting critical thinking among them. By means of inquiry-based learning, students participate in real scientific exploration, under which they must

create hypotheses, evaluate data, and arrive at conclusions—all enhanced by mathematical competency. Examining experimental results shows this relationship especially clearly since validating scientific claims and interpreting results depend on statistical approaches and mathematical modeling. Moreover, using mathematical ideas in scientific settings helps students to better understand both fields and shapes their attitude toward creativity and problem-solving. Teachers are thus urged to create courses that highlight this integration, in line with recent studies supporting a more cohesive framework in STEM education, so ensuring that students emerge as skilled problem solvers able of negotiating the complexity of the modern scientific environment (Jimenez-Aleixandre et al., 2010; Warner et al., 2018).

Mathematical thinking has a significant influence on student involvement, especially in scientific education using inquiry-based learning structures. By helping students to better grasp scientific ideas, involving mathematical ideas supports critical thinking. Studies demonstrating audience response systems and guided inquiry worksheets have improved engagement and learning outcomes across STEM disciplines show These instruments help students to communicate their knowledge and apply mathematical reasoning in related problems, so promoting a cooperative learning environment.

This mathematical approach greatly affects student involvement by encouraging active participation in learning environments and closer knowledge of the subjects. Including mathematical reasoning activities in different courses—such as computer science and software engineering—has improved student involvement by offering real-world, relevant examples that pique interest and inquiry (Hollingsworth & Sitaraman, 2014). Theoretical models including self-determination theories and expectation-value help to clarify how participation in mathematics might be seen both personally and systemically, so stressing the need of personal relevance and contextual elements in encouraging engagement (Watt & Goos, 2017). Empirical studies show that instructional strategies combining cooperative learning with metacognitive training greatly improve students's mathematical reasoning and engagement, surpassing other instructional approaches in graph interpretation and construction (Kramarski & Mevarech, 2003). Furthermore, the relevance of mathematics to personal and society life and its part in developing critical thinking and decision-making skills highlights its possibility to involve students by linking mathematical ideas to practical uses (FitzSimons, 2003). Emphasizing student reasoning and active participation, inquiry-oriented education helps students to investigate mathematical ideas thoroughly and cooperatively, thus improving their knowledge and appreciation of mathematics (Andrews-Larson et al., 2021). These results imply that by making learning more relevant and interactive when properly included in instructional strategies, mathematical thinking can greatly increase student engagement.

However, as students choose alternative courses to maximize their Australian Tertiary Admissions Ranking (Hine, 2017) shows, problems, including student reluctance to pursue advanced mathematics due to perceived relevance and difficulty, can compromise engagement. Therefore, removing these obstacles is essential for teachers trying to create a setting where mathematical reasoning can interact with scientific research to enliven student involvement and support critical thinking.

4. CRITICAL THINKING SKILLS DEVELOPED THROUGH IBL

Encouragement of scientific literacy among students depends first on developing critical thinking skills via IBL. This pedagogical method motivates students to actively solve problems, so fostering analysis, questions, and conclusions derived from their investigations. These instruments evaluate students' knowledge and challenge them to apply mathematical reasoning to new situations, so strengthening their conceptual understanding (Koretsky et al., 2018). Moreover, as the research on teacher education and curriculum development across Europe (Jimenez-Aleixandre et al., 2010) notes, good argumentation techniques inside IBL systems help students to create coherent arguments and critique others' reasoning. In the end, IBL creates an environment where critical thinking abilities may grow, so producing competent scientific research.

With its definition spanning many elements that enable deeper knowledge and problem-solving, critical thinking is essentially connected to the inquiry-based learning method in science. Educational literature defines critical thinking as a process of methodically synthesizing knowledge, analysis, and evaluation that develops an attitude committed to research and introspection. In scientific settings, where mathematical thinking is crucial in developing and testing hypotheses, this ability set is absolutely vital.

While conventional learning models sometimes present knowledge as a fixed entity, critical thinking encourages students to actively interact with material, so challenging assumptions and investigating other points of view. As underlined in recent debates on educational technology, good integration of these elements can redefine learning opportunities and turn students from simple information consumers into active constructors of knowledge (Warner et al., 2018). In the end, encouraging creative problem-solving skills mostly depends on developing critical thinking, especially in the scientific field (Jimenez-Aleixandre et al., 2010).

In scientific education, where mathematical thinking is a fundamental tool, the relationship between IBL and critical thinking abilities is especially clear-cut. By means of IBL, students actively construct knowledge, so improving their analytical capacity and problem-solving ability. By encouraging an environment where students actively participate in questioning, exploration, and knowledge construction, inquiry-based learning greatly improves critical thinking abilities in many different learning environments. Emphasizing the need of thorough teacher preparation and supportive learning environments to maximize its advantages, the systematic review by Sam shows that IBL enhances critical thinking, motivation, and academic performance (Sam, 2024). In music education, Costes-Onishi notes a gap in research activity that might foster critical thinking and suggests that IBL might balance the development of music skills and critical thinking dispositions (Costs-Onishi, 2022). Kelly Y. L. Ku et al.'s study shows that combining IBL with direct instruction improves critical thinking performance, implying that a mixed instructional approach might be more successful than a single method (Ku et al., 2014). < Moreover, the argumentative-based inquiry (ABI) approach—as investigated by Esra Kabataş Memiş and Büşra Nur çakan Akkaş—showcases notable gains in critical thinking skills among students, so demonstrating the value of organized inquiry activities in science education (Memis & Akkas, 2020). Finally, the multidisciplinary research by Daniela B. Friedman et al. emphasizes the flexibility of IBL across several fields, with favorable student comments stressing its possible to enhance higher education experiences (Friedman et al., 2010). These studies confirm that, given enough support and customized to the particular educational setting, IBL is a strong pedagogical tool for developing critical thinking skills.

Notwithstanding the possible benefits of these pedagogical strategies, some challenges still exist; students sometimes complain about mathematics and view other academic paths as more suited. This view could reduce the general efficiency of IBL in fostering critical thinking since students might ignore the intrinsic worth of mathematical ideas for scientific research. Dealing with these issues and promoting a more favorable attitude toward mathematics as a fundamental component of scientific literacy is crucial if one realizes the advantages of IBL completely.

Fostering student involvement and conceptual understanding in scientific disciplines depends on evaluating critical thinking in settings of inquiry-based learning. Studies show that tools like audience response systems help this process work since they involve students actively and let teachers properly evaluate their knowledge (Koretsky et al., 2018). The different uses of these tools in engineering and biology courses especially highlight the need for context-specific evaluation techniques. By contrast, engineering stressed using past knowledge to solve challenging problems, fostering higher-order thinking skills; biology used guided inquiry worksheets for basic understanding. Furthermore, matching research methods with metacognitive competencies, which remain vital for teacher education and professional development, including argumentation theory into assessment frameworks, helps to develop critical thinking further (Jimenez-Aleixandre et al., 2010). Therefore, deliberately evaluating critical thinking in these contexts can greatly improve the students' scientific education results.

5. DISCUSSION

Including IBL in science education changes pedagogy drastically and emphasizes a student-centered approach that develops critical thinking using mathematical reasoning. This change forces teachers to create courses promoting inquiry and experimentation and include mathematical ideas as tools for scientific event analysis. Consistent with the main objectives of IBL, evidence from the Creative Little Scientists project emphasizes the need of combining science and mathematics to improve children's creativity and involvement in learning environments (Stylianidou et al., 2018). Moreover, the shift toward constructivist teaching strategies—as observed in recent engineering education projects—emphasizes the critical need of group learning and emotional intelligence in fostering critical thinking, particularly for underrepresented

groups (Arce-Trigatti, 2020). Therefore, scientific education strategies have to change with these revelations to create a whole learning environment that promotes analytical abilities and emotional involvement to equip students for challenging problem-solving situations.

Examining the subtle interaction between these pedagogical approaches and the improvement of critical thinking skills is essential in considering the future directions for research in IBL and mathematical reasoning. Future research should give top priority to longitudinal studies evaluating the long-term effects of IBL systems on mathematical reasoning capacity of students, so clarifying the cognitive mechanisms behind successful problem-solving. Moreover, multidisciplinary projects could produce creative approaches combining IBL techniques with digital technologies so enabling a more active involvement in STEM education. Furthermore, focus on different student populations can provide important new perspectives on how mathematical thinking develops in IBL environments under influence of cultural surroundings. Future studies can build a more strong theoretical framework that defines the symbiotic interaction between IBL and mathematical reasoning by addressing these several aspects, so promoting critical thinking skills necessary for students' success in ever more complicated and dynamic scientific environments.

Including IBL into science education drastically changes pedagogy by stressing a student-centered approach that develops critical thinking by mathematical reasoning. This change calls for teachers to create courses that support inquiry and experimentation as well as include mathematical ideas as tools for examining scientific events. Consistent with the main objectives of IBL, evidence from the Creative Little Scientists project emphasizes the need of combining science and mathematics to improve children's creativity and involvement in learning environments (Stylianidou et al., 2018). Moreover, the shift toward constructivist teaching strategies—as observed in recent engineering education projects—emphasizes the critical need of group learning and emotional intelligence in fostering critical thinking, particularly for underrepresented groups (Arce-Trigatti, 2020). Therefore, scientific education strategies have to change with these revelations to create a whole learning environment that promotes analytical abilities and emotional involvement to equip students for challenging problem-solving situations.

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The findings of this study underscore the significant role of mathematical reasoning in supporting critical thinking within inquiry-based learning (IBL) environments. This synergy not only deepens students' understanding of scientific phenomena but also equips them with the cognitive tools necessary for tackling complex problems. The implications of these findings are multifaceted and have the potential to influence both academic research and practical applications in science education.

One of the most salient contributions of this study is its alignment with contemporary educational frameworks, such as the Next Generation Science Standards (NGSS). By emphasizing the integration of mathematical reasoning in IBL, the research provides actionable insights for curriculum developers seeking to create interdisciplinary and student-centered learning environments. Practitioners, including educators and instructional designers, can draw upon these findings to design pedagogical strategies that foster higher-order thinking skills. For example, incorporating mathematical reasoning tasks into science curricula can help students build robust analytical frameworks, thereby enhancing their problem-solving capacities. Moreover, the study's findings have implications for teacher professional development. Providing educators with training on integrating mathematical reasoning into IBL can enhance their ability to facilitate student learning effectively. This, in turn, can lead to improved educational outcomes, particularly in STEM fields.

Policymakers may also find this research valuable for developing educational policies that promote critical thinking and interdisciplinary learning.

Although the study provides useful insights, there are a number of limitations that should be taken into consideration. The majority of the research is conceptual, and there is still a limited amount of empirical validation of the framework that has been proposed. Furthermore, the focus of the study was on theoretical integration, which may not have considered all of the various contextual factors that influence the implementation of project-based learning (IBL). These factors include cultural differences, the availability of resources, and different levels of teacher expertise. There are also limitations imposed by the size of the sample and the scope of the study. In the context of inquiry-based learning (IBL), a more comprehensive understanding of the interaction between mathematical reasoning and critical thinking could be obtained through a more extensive empirical investigation that involves a variety of educational settings and student populations. Furthermore, in order to improve the generalizability and applicability of the findings, it is recommended that future research address these limitations.

Building on the current findings, future studies should explore the following areas:

- 1. Empirical Validation: Conduct longitudinal studies to assess the impact of integrating mathematical reasoning into IBL on student learning outcomes.
- 2. Contextual Adaptations: Investigate how cultural and contextual factors influence the effectiveness of IBL frameworks that emphasize mathematical reasoning.
- 3. Technological Integration: Explore the role of digital tools and technologies in facilitating the integration of mathematical reasoning in IBL environments.
- 4. Teacher Training Programs: Examine the efficacy of professional development programs designed to equip educators with the skills to implement IBL effectively.
- 5. Cross-Disciplinary Applications: Investigate the potential for applying the findings to other disciplines, such as social sciences and humanities, to foster critical thinking.

6. CONCLUSION

Combining inquiry-based learning with mathematical reasoning in science education greatly improves critical thinking. Especially in elementary school environments, teachers can help students actively participate in cooperative inquiry so that they develop important analytical skills during formative educational phases. This pedagogical method helps students to grasp scientific ideas more deeply and provides the cognitive tools they need to solve problems successfully. As the report on argumentation in education points out, reformulating courses and teacher training programs to more fully include these approaches is still in great demand. Furthermore, as the analysis of Chinese mathematics textbooks emphasizes, well-crafted instructional models can use contextualized learning and efficient communication to support higher-level cognitive abilities. Thus, giving research-based learning combined with mathematical thinking top priority will significantly affect students' academic paths and capacity for lifelong learning.

Analyzing IBL in science shows that developing critical thinking abilities in students depends critically on mathematical thinking. Important results show that IBL efficiently involves students in challenging problem-solving, which calls for the use of mathematical ideas to evaluate information and reach decisions. As the paper on argumentation theory for teacher education describes, strong pedagogical strategies are absolutely vital in teaching IBL. Many educational systems, meanwhile, fall short in fully including such approaches across courses. Moreover, students' dislike of advanced mathematics courses, underlined in studies on secondary education issues, emphasizes the need to establish favorable learning environments that maintain involvement in higher-order mathematical reasoning and inquiry. By bridging these gaps, the educational structure can be improved, and students will be able to negotiate scientific questions with the critical analytical tools needed for both academic and career success.

The research highlights the transformative potential of combining inquiry-based learning with mathematical reasoning in order to foster critical thinking in the field of science education. The combination

of these pedagogical approaches not only improves the cognitive capabilities of students, but it also ensures that they are adequately prepared to face the challenges that come with living in a world that is becoming increasingly knowledge-driven and interdisciplinary.

In light of these findings, it is imperative that educational stakeholders make the incorporation of mathematical reasoning into IBL frameworks a top priority. When educators do this, they are able to create learning environments that are not only intellectually rigorous but also engaging for their students. The implementation of this strategy is in line with the educational priorities of the present day, which include the promotion of STEM literacy and the preparation of students for careers in science and technology.

To advance research in this field, the following recommendations are proposed:

- 1. Methodological Refinement: Researchers should adopt mixed-methods approaches to capture the multifaceted nature of IBL and its impact on critical thinking.
- 2. Adoption of Advanced Technologies: Leveraging tools such as virtual labs and adaptive learning platforms can enhance the effectiveness of IBL and provide richer data for analysis.
- 3. Addressing Challenges: Researchers should focus on overcoming specific barriers to IBL implementation, such as resource constraints and teacher readiness.

Additionally, the following recommendations that can be put into action are proposed for the stakeholders:

- 1. Practitioners: Educators should design and implement lesson plans that incorporate both inquiry-based activities and mathematical reasoning tasks. For example, physics teachers can use real-world problems requiring mathematical modeling to enhance student engagement and understanding.
- 2. Policymakers: Develop policies that support interdisciplinary approaches in education, emphasizing the integration of mathematics and science to foster critical thinking.
- 3. Researchers: Identify gaps in the literature and design studies to address unresolved questions, such as the long-term effects of IBL on critical thinking skills.

Despite of the contributions it makes, the study reveals a number of areas that have not been uncovered. For example, it is not yet known how it will affect the students' career paths in the long run when they participate in IBL. In a similar vein, the role that dynamics between teachers and students play in determining the success of IBL projects should be investigated further. In order to achieve educational objectives, it is necessary to address these gaps in order to obtain a more nuanced understanding of how IBL can be optimized.

In conclusion, the findings of this study should serve as a rallying cry to education professionals, researchers, and policymakers to recognize the potential of inquiry-based learning (IBL) and mathematical reasoning as potent tools for the development of critical thinking skills. An educational landscape that is more effective and equitable can be contributed to by stakeholders if they address the gaps that have been identified and put the recommendations that have been proposed into action.

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