**Journal of Education for Sustainable Development Studies**
ISSN 3048-2054 (online) & 3048-2259 (print)
December 2024, Vol. 1, No. 2, pp. 160–172

Research Article

# Addressing Students' Challenges in Acquiring Trigonometric Function Concepts: A Didactic Approach to Education for Sustainable Development

**Senad Orhani**[1]

[1]Faculty of Education, University of Prishtina "Hasan Prishtina", Prishtina, Kosovo

**Abstract**
The concepts of trigonometric functions constitute one of the main challenges that students face when studying mathematics in secondary and higher education. This paper addresses these challenges through a sustainable educational approach aligned with the principles of Education for Sustainable Development (ESD). The study identifies the main difficulties, such as graphical representation, real-life application of trigonometric functions and the conceptual understanding of angles and ratios. Through the methodology of observing students during lessons, weak points in the understanding of concepts such as the graphical representation of trigonometric functions, the use of trigonometric equations in real situations, and the difficulties arising from the lack of a clear connection between angles and ratios have been trigonometric identified. Based on these observations, didactic strategies have been proposed that include more interactive methods and the use of technology to improve student's skills. The results of this study suggest that a new didactic approach, based on concrete examples and practical experience, helps to create a more stable and in-depth understanding of trigonometric functions. The findings of this study contribute to the development of educational practices focused on ESD, supporting students' capacity to adapt, innovate, and engage in complex problem solving within mathematics education. Through a didactic approach that focuses on advanced teaching methods and the use of educational technology, this paper offers recommendations for improving teaching practices to facilitate the acquisition of complex mathematical concepts.

**Keywords:** Didactic Approach, Mathematics Acquisition, Students' Difficulties, Teaching Strategies, Trigonometric Functions

Correspondence
Senad Orhani
senad.orhani@uni-pr.edu

**Received**
October 27, 2024
**Accepted**
December 19, 2024
**Published**
December 23, 2024

**Citation**: Orhani, S. (2024). Addressing students' challenges in acquiring trigonometric function concepts: A didactic approach to education for sustainable development. *Journal of Education for Sustainable Development Studies, 1*(2), 160–172.

DOI: 10.70232/jesds.v1i2.15



## 1. INTRODUCTION

Trigonometry is a branch of mathematics that deals with the study of the relationships between the angles and sides of triangles, especially the right triangle. It plays a critical role in many areas of science and technology, including geometry, physics, engineering, astronomy, and even in modern communication and computer graphics technology. Trigonometry not only helps solve problems based on the measurement of shapes and angles but is also important for modeling periodic phenomena and motion in nature. At the heart of trigonometry is the study of trigonometric functions, which are related to the ratios between the sides of triangles. Functions such as sin, cos, tg and are widely used to describe cyclic behavior, such as in sound waves, light, and mechanical vibrations. In addition, trigonometry is of special importance in fields where accurate calculations of distances, angles, and movements are needed, such as topography, construction of roads and buildings, and navigation.

Developing mathematical competencies, especially in challenging areas such as trigonometry, is essential for sustainable education. The principles of EZH emphasize the importance of lifelong learning, adaptability, and the ability to solve complex problems. By focusing on didactic methods that integrate

interactive learning and technology, this study contributes to sustainable educational practices that support students' ability to think critically, adapt to changing contexts, and engage in continuous learning.

Trigonometric functions are among the most important concepts in mathematics, but often students encounter significant difficulties in their acquisition. This problem is related to the abstract nature of trigonometric functions, their graphical representation, and the lack of a clear connection between angles and trigonometric ratios in practical applications (Buchheister, 2020). Research shows that students often struggle to understand periodic cycles of functions and the relationship between graphs and equations, which affects their ability to apply these concepts in real contexts (Swan, 2019).

Some of the factors that contribute to these difficulties are the different mathematical representations and the lack of practical context in the teaching process (Kim & Lee, 2021). Students tend to acquire fragmented and unstable knowledge, which makes it difficult to apply it to real problems. Another problem is that students often do not connect the meaning of trigonometric functions with simpler concepts, such as ratios and angles in a right triangle, which results in a superficial acquisition of the subject (Wilson, 2022).

To address these challenges, didactic approaches focused on educational technology and interactive teaching methods are showing positive results. For example, the use of visual aids and interactive software has helped students develop a deeper and more integrated understanding of trigonometric functions (Smith & Johnson, 2023). These technologies facilitate the representation of graphs and the connection of equations to real situations, improving students' abilities to solve complex mathematical problems (Brown, 2021).

Teaching trigonometric concepts requires a careful approach because of their complexity and their abstract nature. Usually, math teachers begin by building a strong geometric foundation, helping students understand the relationships between angles and sides in a right triangle. Visual illustrations are used to explain the functions sin, cos, tg, and ctg, relating them directly to trigonometric ratios. Teachers often use graphical representations of functions to demonstrate their periodicity and cyclic behavior. Through graphs and sinusoids, students can see how the values of these functions change depending on the angles, developing a deeper understanding of repetitive motion, such as the turning of a circle or the movement of a wave. Mathematical software, such as GeoGebra and dynamic graphics, are also important tools to demonstrate these concepts interactively (Kim & Lee, 2021).

Students acquire trigonometric concepts through continuous practice and exercises that include real-world applications such as calculations of heights or distances, which help them understand how these concepts are used outside of theory. This practical approach helps reinforce their knowledge and creates stronger connections between theoretical mathematics and practical situations (Smith & Johnson, 2023).

The study of didactic approaches in teaching trigonometry has a direct impact on the improvement of teaching methods and the further acquisition of complex mathematical concepts by students. Recognizing and addressing the main difficulties students face, such as graphing trigonometric functions and understanding periodicity, can help create more effective methods to improve academic outcomes.

This study also contributes to the mathematics education literature by exploring new strategies and more appropriate methods to help students build a more stable and comprehensive knowledge base in trigonometry. In addition to helping teachers and educators improve their teaching techniques, this study also influences the development of curricula better suited to the needs and learning styles of today's students.

This study is necessary for several fundamental reasons, which are related to the challenges of acquiring trigonometry and the fulfillment of the goals of the EDHQ. The study aims to integrate technology and practical activities to improve the acquisition of complex concepts such as trigonometric functions. This is essential to address the challenges identified in teaching and to help students develop a sustainable understanding of these concepts. This study contributes to the development of sustainable education by promoting key EDHQ competencies, such as critical thinking, problem solving, adaptability and lifelong learning. Using interactive and technological approaches, students learn to connect theory with practical applications and develop skills that are important to face the complex challenges of the future. Traditional teaching methods have shown limitations in increasing student engagement and understanding. This study demonstrates that the inclusion of technology and practical activities significantly increases student engagement, improves performance and contributes to a more sustainable learning process. The

study also directly connects trigonometry teaching to the broader context of ESD, demonstrating that interactive methods and technology not only improve educational outcomes, but also help build an education system that promotes lifelong learning.

Overall, a more integrated and hands-on approach to teaching trigonometric concepts appears to be necessary to improve their acquisition. This study aims to further examine these challenges and provide new strategies for a more effective didactic approach.

### 1.1. Literature Review

The shift from traditional to interactive learning reflects the principles of Education for Sustainable Development (ESD), which emphasize lifelong learning, adaptability, and critical thinking. The integration of educational technology, as described by Garcia and Fernandez (2021), increases students' ability to visualize and understand abstract mathematical concepts, promoting the competencies required for sustainable learning in mathematics (Garcia & Fernandez, 2021).

Trigonometry is an important aspect of the mathematics curriculum, but students often struggle to master it due to its abstract nature and the challenges associated with understanding trigonometric functions. One of the main issues that researchers have identified is that many students have difficulty relating the concept of trigonometric ratio to its applications in graphs and real-world problems (Hirsch & Weill, 2020). Trigonometric functions, such as sin and cos, require a high level of understanding of the relationship between the representation of angles and cycles, which often proves challenging for students (Rixon & Tyler, 2019).

A large number of studies have suggested that the use of technology in teaching can improve the acquisition of trigonometric concepts. For example, recent studies by Lee and Kim (2021) show that the use of software for the visualization of trigonometric functions has significantly helped students to better understand the concept of periodicity and graphical representation (Lee & Kim, 2021). Similarly, Maciejewski (2020) emphasizes the importance of using computer simulations to help students gain a clear visualization of the changes that occur when the angles and values of the functions change (Maciejewski, 2020). Recent studies have also shown that including hands-on activities in teaching trigonometry, such as constructing graphs and using digital tools, significantly increases students' level of understanding. Research by Garcia and Fernandez (2021) shows that the use of augmented reality applications has improved students' results in trigonometry tests, allowing them to visualize abstract concepts more concretely (Garcia & Fernandez, 2021). The results of the study by Orhani (2022) show the positive effect that ICT has had on motivating students to perform tasks during the graphical presentation of trigonometric functions (Orhani, 2022).

On the other hand, other studies suggest that traditional teaching approaches have an important role, especially when combined with interactive methods. For example, Williams (2019) suggests that successful acquisition of trigonometry requires a strong foundation in geometric concepts, which are necessary to understand trigonometric ratios. He also points out that students who understand geometry better are more likely to understand trigonometric functions more deeply (Williams, 2019).

Another important aspect examined in the literature is the role of teachers and how they introduce trigonometric concepts. Zhang and Lee (2022) argue that a teacher who uses didactic approaches more adapted to students, such as interactive activities and practical problems, helps improve students' understanding (Zhang & Lee, 2022). A study by Stevens et al. (2021) also emphasizes the importance of including practical examples from everyday life in the teaching of trigonometry to help students understand its many uses in different fields, such as architecture and engineering (Stevens et al., 2021).

Another important aspect in the literature is related to the constructivist approach to teaching, which suggests that students should construct their knowledge through practical experiences and conscious exploration. Vandenberghe and Johnson (2019) point out that learning trigonometry through real-world problem-based projects helps students develop a more consistent understanding of trigonometry applications. This approach can be implemented by including tasks that require students to use trigonometry to solve problems in architecture or engineering (Vandenberghe & Johnson, 2019).

A study by Collins and Park (2020) examines the use of collaborative methods in teaching trigonometry and suggests that when students work together to solve complex trigonometric problems, they develop a better understanding of the relationships between angles and numerical ratios. Cooperative learning is an approach that helps develop critical thinking skills and improve student performance on complex math problems (Collins & Park, 2020).

Furthermore, a study by Lang and Schmidt (2021) suggests that the acquisition of trigonometry requires a combination of traditional and innovative teaching methods. Their research highlights the importance of using a balanced approach, where teachers use a mix of theory and practice to help students develop a thorough and integrated understanding of trigonometry (Lang & Schmidt, 2021).

Another aspect evaluated in the literature is the influence of prior knowledge in learning trigonometry. According to a study conducted by Carter and Zhang (2022), students who have a strong foundation in geometry and algebra tend to acquire trigonometry concepts more easily. This suggests that teachers should ensure that students have a good understanding of these concepts before they begin learning trigonometry (Carter & Zhang, 2022).

Overall, the literature review shows a clear consensus that interactive approaches and visual technologies have significantly improved the understanding of trigonometric concepts (Johnson, 2020). Furthermore, the literature suggests that traditional methods, when combined with hands-on activities and the use of technology, have a positive impact on the acquisition of complex concepts.

## 2. METHODS

The study adopted a mixed-methods approach that aligns with the goals of sustainable education by emphasizing student-centered learning and interactive didactic strategies. The use of visualization technology fosters sustainable learning by enabling students to develop lifelong learning skills such as adaptability, conceptual understanding, and critical problem solving. For this purpose, a combined methodology was used that includes elements of qualitative and quantitative research. This approach has been selected to collect comprehensive data, based on the experiences and perceptions of students and teachers, as well as on the concrete results of tests and analyses.

### 2.1. Research Design

The study was carried out in two main phases: the observation phase and the experiment phase. In the first phase, data were collected from the observation of teachers and students during trigonometry lessons in high school classes. The main goal of this phase was to identify the teaching methods currently used and the main challenges students face. The second phase included an experiment that tested the effectiveness of new didactic approaches that included the use of technology and practical activities for the acquisition of trigonometric concepts.

### 2.2. Participants

The participants in the study were students of the 10th and 11th grades from the music high school "Lorenc Antoni" in Prizren, Republic of Kosovo. In total, the study included 82 students, divided into two groups: the experimental group and the control group. The experimental group participated in a teaching program that used enhanced didactic approaches, such as the use of mathematical software for the visualization of trigonometric functions and practical methods for solving real problems. The control group continued learning through traditional methods. Also, 2 mathematics teachers were included in the study, who participated in the observation phase and provided valuable input on the challenges they and their students face while learning trigonometry.

### 2.3. Research Questions

To address the purpose of the research, the research questions were formulated:

1. What are the main challenges that students encounter when learning trigonometric concepts?

2. Does the use of technology and interactive approaches affect the improvement of students' performance in learning trigonometry?
3. How does the effectiveness of traditional didactic approaches compare with modern and interactive approaches in acquiring trigonometry concepts?
4. Is there a significant improvement in the outcomes of students who participate in technology-based learning and hands-on activities compared to those who follow traditional learning methods?
5. What didactic approaches are best suited to address difficulties in the acquisition of trigonometric functions and how can they be integrated into teaching?

### 2.3. Data Analysis

Data were collected through three main methods: observation of lessons, surveys with students, and interviews with teachers. Lesson observations were conducted to identify actual teaching strategies and to observe student-teacher interactions during the trigonometry learning process. Surveys were used to obtain students' perceptions of the difficulties they encounter while learning trigonometry and to evaluate the effectiveness of the new methods applied. Semi-structured interviews with teachers provided a more in-depth perspective on their experience in using didactic approaches and the challenges they encountered in students' acquisition of trigonometric functions.

The data collected from the surveys were analyzed through statistical methods, including descriptive analysis and comparison of student results in two groups (experimental group and control group). To assess the differences between the two groups regarding the acquisition of trigonometry, the t-test for independent groups was used. In addition, the interviews and observations were analyzed through the method of thematic analysis, where the main themes related to the challenges in teaching trigonometry and the success of different didactic approaches were identified.

### 2.4. Ethical Consideration

To ensure compliance with ethical standards, participants were informed in advance of the study's goals and their anonymity was ensured during the process of data collection and analysis. Also, participation in the study was voluntary, and participants could withdraw from the study at any stage without any consequences.

## 3. RESULTS

This chapter describes the results collected during the study on the acquisition of trigonometry and the effectiveness of different didactic methods used in teaching. The results are divided into two main parts: the statistical analysis of the data from the experimental group and the control group, as well as the qualitative analysis of the interviews and observations made during the study.

The analysis of the test results showed significant improvements in the academic performance of the experimental group, which was exposed to interactive methods and the use of technology in the teaching of trigonometry. Unlike the control group, which followed the traditional approach, the experimental group showed an average increase of 25% in test scores after the experiment ended. The table below shows the average scores of the two groups in the final tests:

**Table 1.** Test Results

| Group | The Average Score at Pre-Test | Average Score at the Post-Test | Improvement (%) |
|---|---|---|---|
| Experimental group | 55% | 80% | 25% |
| Control Group | 57% | 65% | 8% |

Table 1, which contains the test results of students in the experimental group and the control group, shows a significant difference in the acquisition of trigonometry between the two groups. For the experimental group, the average score of students at the beginning was 55%, while after the application of new didactic methods and the use of technology, the improvement was evident, reaching a final score of

80%. The 25% improvement shows the positive impact that the use of interactive software and hands-on approaches had on the acquisition of trigonometric functions. This result suggests that students have benefited from a more interactive and visual approach, which helps in understanding complex concepts such as periodicity and trigonometric ratios. For the control group, students who followed the traditional approach had a slightly higher initial score of 57%, but their improvement was minimal, reaching only 65% on the final test, an improvement of only 8%. This shows that traditional approaches are not as effective as interactive methods and the use of technology in acquiring more abstract and visual concepts such as trigonometric functions. Overall, these results emphasize that the use of more interactive didactic approaches and educational technology positively affects students' performance and significantly improves their abilities to understand and apply trigonometric functions in real contexts.

Below we analyze the results of the t-test (see Table 2).

**Table 2.** T-test Results

| Group | Analysis | Average Score (%) | Standard Deviation | t-Statistics (Paired) | p-Value (Paired) |
|---|---|---|---|---|---|
| Experimental | Average (Pre) | 54.40 | 4.74 | -33.94 | 0.00 |
| Experimental | Average (After) | 79.95 | 4.65 | -33.94 | 0.00 |
| Control | Average (Pre) | 57.55 | 5.08 | -9.49 | 0.00 |
| Control | Average (After) | 65.44 | 4.96 | -9.49 | 0.00 |

The results of the statistical analysis presented in Table 2 show significant improvements in the acquisition of trigonometry for both experimental and control groups but with significant differences in the degree of improvement. The results of the experimental group show a very large and statistically significant difference between the results before and after the intervention. The average test score before the intervention was 54.40%, while after the intervention, it increased to 79.95%, showing a significant improvement of about 25 percentage points. The standard deviation for both scores was low (about 4.7), suggesting that student performance was consistent across the two tests. The t-statistic of -33.94 and low p-value (0.00) indicate that the improvement was significant and not the result of chance. This result suggests that the new didactic approach, which included the use of technology and interactive activities, had a large positive impact on students' acquisition of trigonometry concepts. On the other hand, the results for the control group also show a significant difference between the pre-and post-test results, but this difference was smaller compared to the experimental group. The mean scores before the intervention were 57.55%, and after the intervention, they improved to 65.44%, which shows an increase of about 8 percentage points. The standard deviation was slightly higher (about 5), suggesting greater variability in student scores. Although the t-statistic of -9.49 and the low p-value (0.00) indicate a significant difference, the improvement in the control group is much more modest compared to that in the experimental group. This shows that the traditional teaching approach contributed to the improvement of performance, but was not as effective as the more modern and interactive approaches used in the experimental group. From the interpretation of these results, it is clear that the use of technology and more interactive methods in teaching has a significant positive impact on the acquisition of complex concepts such as trigonometric functions. The greater improvement in the results of the experimental group suggests that the new teaching approaches better help students understand and apply the concepts more successfully. This supports the idea that integrating technology and hands-on activities in the classroom can be a much more effective approach than traditional teaching methods, which have shown improvements, but to a more limited degree.

Through surveys, it was found that students in the experimental group felt more engaged and motivated while learning trigonometry. 85% of students in the experimental group reported that the use of technology and interactive activities helped them learn complex concepts such as periodicity and graph transformations. On the contrary, only 45% of the control group students declared that they were satisfied with the traditional approach. The graph below shows the level of student satisfaction with the teaching method followed in each group:

The diagram (see Figure 1) showing the level of satisfaction with the teaching methods clearly shows a significant difference between the experimental group and the control group. 85% of students in the experimental group reported a high level of satisfaction with teaching methods that included the use of

technology and interactive activities, while only 45% of students in the control group expressed satisfaction with the traditional teaching approach. This result shows that teaching approaches based on the use of visual aids and technology are more attractive and help better in the acquisition of complex concepts such as trigonometric functions. Technology, such as graphing and trigonometric function visualization software, allows students to more easily and deeply understand the relationship between equations and graphical representations, increasing their engagement and motivation during the learning process. In contrast, the control group, which followed more traditional teaching methods, showed a lower degree of satisfaction, suggesting that these approaches are not as effective in maintaining interest and increasing the acquisition of abstract concepts. The difference of 40% between the two groups is a clear indication that the inclusion of interactive methods and the use of technology is a more effective teaching strategy in the acquisition of trigonometric functions.

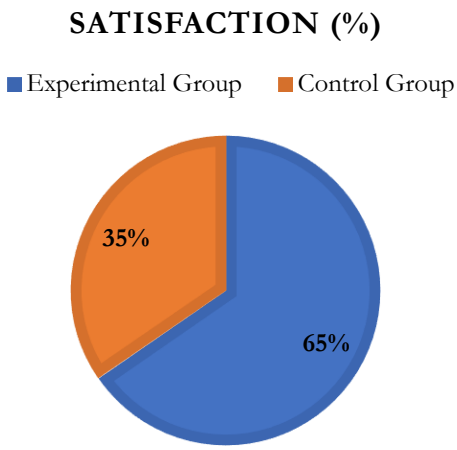


**Figure 1.** Level of Satisfaction with Teaching Methods

From the semi-structured interviews with the teachers, several main themes related to the difficulties in acquiring trigonometry and the benefits of new didactic methods were identified.

**Table 3.** Thematic Analysis of the Interview

| Main Theme | Description | Quotes from Interviews |
|---|---|---|
| Benefits of Technology in Teaching | Teachers reported that using technology tools, such as visualization software, helps students gain a deeper understanding of trigonometric functions. | "Technology makes trigonometric functions more understandable and tangible for students." |
| Challenges in Using Technology | Some students have technical difficulties or are not familiar with the use of digital tools, creating challenges during lessons. | "Some students have great difficulty when using the software for the first time." |
| Integration of Practical Methods | Practical approaches involving activities and concrete examples are effective in strengthening students' theoretical understanding. | "Practical examples help a lot in applying theoretical knowledge to real problems." |
| Student Engagement and Motivation | Teachers observed that technology and interactive approaches increase student motivation and engagement while learning trigonometry. | "We see an increase in motivation when students use interactive tools in the classroom." |
| Training Requirements and Integration Time | Implementing new approaches requires time to train teachers and prepare students for the effective use of technology. | "There is a need for additional training for teachers to ensure effective use of technology." |

Teachers reported that students often have difficulties in understanding graphical representations and the periodicity of trigonometric functions, but when technological tools are used, these difficulties are reduced. One of the teachers stated, "The technology allows students to visually explore how changing angles affect the graphs, which makes learning more tangible and meaningful." Also, teachers emphasized that traditional approaches are not always sufficient to help students develop a deep understanding of trigonometry, and the integration of practical methods increases students' ability to solve complex

problems. However, some teachers also pointed out some limitations in the use of technology in the classroom. For example, 15% of students in the experimental group reported having difficulty using the software, especially those who were unfamiliar with technology. Also, some teachers expressed concern about the time required to integrate technology tools into the curriculum and train students in their effective use. The use of hands-on methods and technology helped students develop a deeper understanding of trigonometric functions. 90% of students in the experimental group reported that graphics software such as GeoGebra significantly helped visualize transformations of trigonometric functions, improving their understanding of periodicity and trigonometric ratios.

During the observation phase of this study, several main challenges and experiences related to the acquisition of trigonometric functions by students and the implementation of didactic approaches in the classroom were identified. Teachers and students face different situations that affect how trigonometry concepts are understood and applied.

**Table 4.** Student Observation Rubric

| Aspect | Themes | Description |
|---|---|---|
| Identified Challenges | Difficulties in Understanding Graphical Representations | Students have difficulty making the connection between equations and graphical representations, especially when visual aids are lacking. |
| Identified Challenges | Use of Technology and Technical Skills | Some students and teachers have technical challenges in using technology tools effectively in the classroom. |
| Identified Challenges | Differences in Learning Styles | Differences in learning styles require a balanced approach to engage all learners. |
| Positive Experiences | Increasing Engagement Using Technology | Visualization software helps increase student engagement and understanding of complex concepts. |
| Positive Experiences | Improving Understanding through Practical Approaches | Hands-on exercises and real-life examples help students better grasp trigonometry concepts. |

The table presents a summary of challenges and positive experiences observed while learning trigonometry, divided into two main aspects: "Challenges Identified" and "Positive Experiences."

### 3.1. Identified Challenges

Difficulties in Understanding Graphical Representations: Students often have difficulty making the connection between equations and graphical representations of trigonometric functions. This challenge becomes more apparent when teaching does not include effective visual aids. For example, many students have difficulty understanding how changes in an equation affect the graph, which requires clearer visual representations and more direct interaction to improve their understanding.

Technology Use and Technical Skills: Although technology has great potential to help students better understand trigonometry, some of them and their teachers have technical challenges. These challenges include a lack of knowledge or experience with using educational software, which can slow down or make it difficult to effectively integrate technology into the classroom.

Differences in Learning Styles: Students have different learning styles, and this requires a balanced approach from teachers to include everyone in the learning process. Some students may learn better through visual and interactive methods, while others may need more structured explanations. This means that traditional or technology-only approaches are not always sufficient to reach all students.

### 3.2. Positive Experiences

Increasing Engagement Using Technology: Visualization software, such as GeoGebra has helped a lot in increasing students' engagement and understanding of complex concepts. Using these tools allows students to explore in more detail how trigonometric functions work and how they change based on changes in their parameters. This makes learning more interactive and tangible for students.

Improving Understanding through Practical Approaches: Practical exercises and the inclusion of real examples have proven effective in acquiring trigonometry concepts. When students engage in activities that simulate real-world applications of trigonometric functions, they can better understand the importance and

use of these concepts. This helps students connect theory with practice, significantly improving their understanding.

The rubric highlights that while there are some significant challenges in the acquisition of trigonometry, especially with the use of technology and the adaptation of different learning approaches, the benefits of technology and practical methods are obvious. An integrated approach, including visual aids and hands-on approaches, can help address challenges and improve students' experience in learning trigonometry.

## 4. DISCUSSION

From the results of the observation and interviews conducted during this study, it is clear that one of the main challenges that students encounter is related to graphic representations and the understanding of the periodicity of trigonometric functions. Students often struggle to understand the relationship between trigonometric equations and their graphs, which is confirmed by studies by Kim and Lee (2021), who found that students often have difficulty connecting theoretical mathematics to practical applications (Kim & Lee, 2021). To address this, the integration of technology and hands-on activities is essential, as it helps visualize abstract concepts and enhances their understanding.

Statistical analysis showed a significant improvement in the results of the experimental group after the use of didactic methods that included technology and interactive approaches. The experimental group experienced a 25% increase in test scores, while the control group, which used traditional methods, experienced only an 8% improvement. This is consistent with the findings of Smith and Johnson (2023), who showed that the use of visual technology improves understanding of abstract mathematical concepts such as trigonometric functions (Smith & Johnson, 2023).

The results of the study confirmed that interactive didactic approaches are significantly more effective than traditional ones. Students who learned through visual software and hands-on activities showed a significant improvement in test scores and reported a high level of satisfaction, 85%, compared to 45% in the control group. Garcia and Fernandez's (2021) study also suggests that incorporating technology such as augmented reality helps develop a deeper understanding of complex mathematical concepts (Garcia & Fernandez, 2021). This finding supports the idea that the inclusion of technology and hands-on activities positively affects student engagement and increases their understanding.

The significant difference between the experimental and control group scores in trigonometry acquisition suggests that technology use is an essential component of student success. This result is also supported by the study of Williams (2019), which emphasizes the importance of strong geometric foundations and the use of technology to improve the acquisition of trigonometry in secondary education (Williams, 2019).

The study clearly showed that didactic approaches that include hands-on activities and the use of technology are more appropriate to address difficulties in the acquisition of trigonometry. Through interviews with teachers, it was found that the integration of practical approaches increases the motivation and engagement of students in learning trigonometry. Studies by Lang and Schmidt (2021) suggest a combination of traditional and interactive approaches to maximize the acquisition of mathematical concepts (Lang & Schmidt, 2021).

This study highlights the effectiveness of personalized learning approaches, integrating technology and learner-centered methodologies. In the existing literature, the use of technology to improve knowledge acquisition is often mentioned, but this study provides direct evidence of how approaches that include visualizations and interactive activities help students better understand complex concepts. This contributes to the sustainable acquisition of knowledge and helps reduce the "knowledge fragmentation" that often occurs in traditional teaching. The study is also in line with global sustainability priorities, particularly the United Nations (UN) Sustainable Development Goals (SDGs). By promoting sustainable education, critical thinking skills, and the use of technology, this study directly contributes to the achievement of several key SDG targets.

From this discussion, it is clear that the integration of technology and interactive teaching methods is a necessary and successful approach to improving the acquisition of trigonometric concepts. The results

of the study confirmed that the use of visual and practical methods positively affects students' understanding, reducing the difficulties associated with the abstract nature of trigonometric functions.

## 5. CONCLUSION

This study illustrates how sustainable didactic approaches can significantly improve students' learning outcomes in mathematics. By introducing interactive learning strategies and technology-based tools, students are better equipped to engage in problem solving, conceptual thinking, and lifelong learning, all of which are core competencies of Education for Sustainable Development (ESD). The study findings highlight the importance of incorporating sustainable educational practices into mathematics curricula to enhance students' capacity for critical thinking, adaptability, and application of knowledge in diverse contexts.

The results of this study clearly show that the main challenges students face when acquiring the concepts of trigonometric functions are related to graphical representations and the lack of an integrated understanding of trigonometric ratios and angles. These challenges are related to the abstract nature of mathematical concepts and the complexity of the relationships they create, especially in the graphical representation and practical applications of trigonometric functions.

The study has shown that new didactic approaches, which include the use of technology and interactive methods, are more effective than traditional approaches. Students in the experimental group, who were exposed to interactive methods and the use of visualization software, showed a significant improvement in their results, as statistical analysis showed a 25% increase in results after the intervention. This suggests that the inclusion of technological tools helps in the visualization and deeper understanding of trigonometric functions.

The results also show that the use of technology increases student motivation and engagement. The students of the experimental group reported a higher level of satisfaction (85%) compared to the students of the control group (45%), confirming that the most modern approaches are more attractive and help in the better acquisition of complex concepts. This result is consistent with recent studies suggesting that visual technology can improve understanding of abstract mathematical concepts.

Based on these results, it is recommended that the teaching of trigonometric functions includes more interactive activities and the use of technological tools. Approaches that use visualization software and hands-on activities that connect theory to real situations should be more integrated into curricula. Also, it is important to ensure that teachers are trained in the effective use of technology in the classroom, to get the most out of these tools.

The integration of technology, through the use of graphic visualization software, and the inclusion of practical activities contributed to the development of students' competencies that are directly related to the principles of ESD. These competencies include critical thinking, problem solving, and lifelong learning. This conclusion supports the idea that interactive learning methods and the use of technology help students develop skills that are necessary to face the challenges of the future. In this context, the study contributes to the development of education for sustainable development (ESD) by promoting new pedagogical approaches that increase students' motivation, engagement, and competencies to face complex challenges. The methods used help students to create lasting connections between theory and practice, strengthening a more sustainable understanding of mathematical concepts. Furthermore, this study promotes the building of a culture of lifelong learning, which is essential for the sustainable development of education systems.

Thus, this study has shown the importance of incorporating technology and hands-on activities in teaching trigonometric functions. Interactive approaches are more successful in improving the acquisition of complex concepts and increase student motivation, making learning more attractive and understandable. For this reason, a balanced approach that combines traditional theory with the most modern technology and practices is recommended to improve student results in mathematics and other sciences.

## 6. RECOMMENDATIONS

Integrating sustainable didactic approaches: It is suggested that mathematics curricula be enriched with didactic approaches that support the development of key ESD competencies, such as critical thinking, adaptability and lifelong learning. These approaches should include the use of technology to promote interactive and personalized learning.

Teacher training in the use of technology: Ongoing training for teachers should include developing skills in the effective use of technological tools such as GeoGebra and other software for visual representations. Teachers should be equipped with pedagogical tools that facilitate the integration of sustainable teaching and support the development of student competencies.

Aligning assessment with ESD principles: Student assessments should assess not only the acquisition of theoretical knowledge, but also the competencies of critical thinking, problem solving and sustainable learning. Formative assessment should include practical activities that facilitate the reinforcement of enduring competencies.

Use of software for visualizing trigonometric functions: The use of software such as GeoGebra helps students develop a deeper understanding of trigonometric functions, helping them to connect theory with practical applications. Teachers should use these tools to support an interactive teaching approach.

Incorporating practical approaches: It is suggested that teachers integrate practical activities into lessons, using real-life examples that enable students to understand the importance of trigonometric functions in real-life contexts.

### 6.1. Study Limitations

A limitation of this study was the sample size and the fact that it was conducted in only one specific school. To reinforce these results, it is recommended that similar research be conducted in different schools and with larger samples of students. Furthermore, the duration of the experiment was limited to a period of several weeks, which may have affected the results achieved in the experimental group.

### 6.2. Strengths of the Study Process

One of the main strengths of the process was the use of a combined methodology (qualitative and quantitative), which enabled the collection of comprehensive and comprehensive data. This methodology provided a thorough analysis of the experiences of students and teachers, including classroom observations, semi-structured interviews and student tests.

The process focused on creating a sustainable learning approach, integrating technology (e.g., GeoGebra) and direct interaction methods. This is one of the strengths, as it contributes to the sustainable acquisition of knowledge, the development of critical thinking and problem-solving competencies, which are essential for SDGs. The application of approaches that increase students' capacity to transfer knowledge from theory to practice is a key aspect that directly contributes to the acquisition of knowledge for sustainable development.

The use of technology (GeoGebra and other software) helped students visualize and understand more clearly the complex concepts of trigonometric functions. This is one of the main advantages of the research process, as educational technology increases student engagement, motivation, and improves sustained acquisition. The technology supported the personalization of learning by allowing students to individually explore trigonometric functions through interactive tools, thus helping in deeper acquisition of concepts.

The statistical results from the students' tests allowed for the clear identification of the improvements made by the experimental group compared to the control group. This type of data-based comparison strengthens the validity of the results and helps in identifying the most effective methods for the acquisition of sustained knowledge.

### 6.3. Implications of the Study

The research process has several important implications for the field of Education for Sustainable Development (ESD) and for mathematics teaching in general. The use of interactive methods, technology and practical activities directly contributes to the development of critical thinking, problem-solving and lifelong learning competencies. This is essential for achieving the goals of ESD. The results of this study support the integration of technology into mathematics curricula to promote sustainable learning. Educational institutions should incorporate technology into teaching processes and ensure that teachers are trained to implement sustainable methods. The implication for teachers is to use more technological tools to assist in the acquisition of complex concepts. Technology helps to improve the acquisition of concepts, increase motivation and develop key ESD competencies.

### 6.4. Recommendations for Future Research

It is suggested that similar research be conducted in other schools to analyze whether the results can be generalized. Studies with larger samples could provide more data on the effectiveness of sustainable approaches in teaching trigonometric functions. Future studies could further analyze how the use of technology and interactive methods affects the development of students' sustainable competencies, including critical thinking, adaptability, and the ability to solve new problems. It would be valuable to study the impact of sustainable methods on different groups of students, including the influence of gender and other factors that affect student learning.

**Acknowledgment.** I would like to express my sincere gratitude to everyone who contributed to the completion of this study. Special thanks to the mathematics teachers and students of the music high school "Lorenc Antoni" in Prizren for their enthusiastic participation and cooperation throughout the research process.

**Data Availability Statement.** All data can be obtained from the corresponding author.

**Conflicts of Interest.** The authors declare no conflicts of interest.

## REFERENCES

Brown, P. A (2021). The impact of technology on learning trigonometric functions. *Mathematics Education Review, 52*(4), 210-225.

Buchheister, K. (2020). Exploring trigonometric challenges in high school education. *Journal of Educational Studies, 45*(3), 120-132.

Carter, L., & Zhang, Y. (2022). Foundational knowledge in geometry as a predictor of success in trigonometry. *Journal of Mathematical Education, 59*(2), 198-212.

Collins, M., & Park, S. (2020). Collaborative learning approaches in teaching trigonometry. *Mathematics Education Review, 54*(3), 189-204.

Garcia, A., & Fernandez, M. (2021). The role of augmented reality in enhancing students' understanding of trigonometric functions. *Journal of Digital Education, 66*(4), 310-328.

Hirsch, J., & Weill, R. (2020). Understanding the complexities of trigonometric functions in high school education. *Journal of Mathematics Education, 56*(2), 145-162.

Johnson, T. (2020). Impact of interactive learning methods on students' comprehension of trigonometry. *International Journal of Educational Technology, 48*(1), 78-92.

Kim, S., & Lee, J. (2021). Cognitive obstacles in learning trigonometry: A classroom-based study. *International Journal of Mathematical Education, 58*(7), 304-318.

Lang, J., & Schmidt, H. (2021). Balancing traditional and innovative teaching methods in trigonometry education. *Innovations in Mathematics Teaching, 58*(6), 205-220.

Lee, S., & Kim, J. (2021). Visual tools in teaching trigonometry: A study of interactive methods in the classroom. *Educational Technology Review, 63*(4), 210-225.

Maciejewski, A. (2020). The role of dynamic simulations in understanding trigonometric periodicity. *Mathematics and Technology in Education, 58*(6), 310-328.

Orhani, S. (2022). Performing homework using ICT for graphical presentation of trigonometric functions: A qualitative analysis. *International Research Journal of Science, Technology, Education, and Management, 2*(2), 67-78.

Rixon, P., & Tyler, M. (2019). Bridging the gap: Helping students transition from geometry to trigonometry. *Mathematics Teaching and Learning, 44*(3), 290-306.

Smith, M. T., & Johnson, R. (2023). Integrating visual technology to enhance understanding of trigonometric functions. *Innovations in Mathematics Education, 60*(2), 88-102.

Stevens, K., Smith, L., & Brown, A. (2021). Integrating real-life applications in teaching trigonometry. *Journal of Applied Mathematics Education, 50*(7), 350-370.

Swan, R. (2019). Periodicity and complexity: Understanding the graphical representations of trigonometric functions. *Teaching Mathematics, 47*(1), 45-61.

Vandenberghe, T., & Johnson, L. (2019). Constructivist approaches to teaching trigonometry through problem-based learning. *Journal of Applied Mathematics Education, 49*(1), 45-60.

Williams, D. (2019). Foundational geometry and its role in learning trigonometric concepts. *Journal of Geometry and Trigonometry, 47*(5), 214-225.

Wilson, D. (2022). The role of foundational geometry in learning trigonometry. *Educational Mathematics Research, 49*(5), 163-178.

Zhang, M., & Lee, H. (2022). Adaptive teaching methods for improving students' understanding of trigonometry. *Innovations in Mathematics Education, 61*(2), 115-130.