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Research Article

A Design-Based Professional Development Module for Enhancing Mathematics Teachers' Calculus Instruction Using GeoGebra

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Abstract
This is a three-week professional growth module for secondary mathematics teachers. The treatment aims to strengthen teachers' ability to teach definite and indefinite integrals by integrating GeoGebra into calculus instruction. Informed by design-based research (DBR) principles, the professional growth module is conceptualized as an evolving instructional system that is refined through repeated implementation, analysis, and redesign, aligning theory, instructional design, and real-world practice. The module draws on constructivist principles, emphasizing learning through interaction, interpretation, and reflection. The instructional design is based on the Technological Pedagogical Content Knowledge (TPACK) framework, highlighting the coordinated integration of disciplinary knowledge, instructional strategies, and digital tools. The module is structured into three scaffolded phases, with Week One dedicated to strengthening teachers' conceptual understanding of integration, particularly the ideas of accumulation and the area under a curve. Week Two emphasizes the development of digital literacy and symbolic skills by engaging teachers in structured GeoGebra explorations, where they dynamically model calculus concepts and connect algebraic, graphical, and numerical representations. Week Three targets instructional competence, requiring participants to design lesson plans, deliver mock teaching sessions with GeoGebra, provide peer feedback, and reflect on their pedagogical approaches. To capture changes in teachers' conceptual understanding, digital literacy, and teaching approaches, data will be generated through pre- and post-Evaluations, reflective surveys, observations, and teaching and learning artifacts. Rather than treating the professional growth module as a static treatment, this study adopts an iterative design-based perspective that allows continuous refinement of both instructional activities and Evaluation tools. By examining teachers' learning across conceptual, technological, and pedagogical dimensions, the study contributes a replicable model for a blended professional growth module for teaching calculus in secondary schools.

Keywords: Blended Learning, Design-Based Research, GeoGebra, Professional Growth

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1. INTRODUCTION

Technology is key in shaping how mathematics concepts are introduced, represented, and explored in classroom settings (Kalu, 2018). In this context, GeoGebra provides an interactive digital platform that allows users to coordinate algebraic expressions, graphical representations, and numerical outputs within a unified workspace (Ardina et al., 2025). Prior research indicates that GeoGebra can substantially shape how mathematical concepts are represented, explored, and discussed in instructional settings. For example, Ardina et al. (2025) investigated GeoGebra integration among mathematics education students in the Philippines and documented increases in students' willingness to participate, their interest in the topic, and their grasp of core concepts. Their findings imply that, when supported by purposeful instructional design, GeoGebra can strengthen learning across a range of mathematical topics. Similarly, Uwurukundo et al. (2020) reviewed 20 studies in Rwanda and found that 80% documented increased student understanding and motivation, highlighting the platform's positive impact. However, a large proportion of existing studies continue to emphasize geometry-focused implementations, leaving areas such as calculus and trigonometry relatively underexplored and suggesting the need for broader investigation. For instance, in Ghana, Benning et al. (2023) found that professional growth programs involving GeoGebra contributed to shifts in both teacher confidence and skill. However, their results underscore how classroom use is strongly influenced by the availability of ongoing assistance, adequate resources, and institutional support. Similarly, Mensah (2025), examining a Swedish context, noted that pre-service teachers tended to adopt a passive role during a GeoGebra-mediated session, suggesting that professional growth formats that actively involve teachers may be more beneficial. Studies further indicate that the usefulness of GeoGebra hinges on teachers' ability to pair the tool with discussion-based and reflective instructional approaches. For example, Zhang (2025) showed that combining GeoGebra-supported visual models with discussion-oriented teaching enhanced pre-service teachers' ability to articulate mathematical ideas. Likewise, Çakır and Alkan (2022) found that pre-service teachers valued GeoGebra's exploratory approach most when it was supported by structured pedagogical guidance.

Nonetheless, notable gaps persist, especially in research that directly addresses the needs of secondary mathematics teachers. Both Ardina et al. (2025) and Benning et al. (2023) stressed the need for sustained, well-supported professional growth programs to bolster teachers' ability to integrate technology and pedagogy effectively. Taken together, these studies point to the necessity of developing a professional growth program specifically tailored to support teachers in advanced mathematics topics, such as integration, where GeoGebra's potential remains underutilized.

1.1. The Current Module

This professional growth module is designed to assist secondary mathematics teachers in blending GeoGebra into the teaching of calculus concepts related to integration. Although procedural competence is common among secondary mathematics teachers, many report insufficient familiarity with the technological and conceptual support necessary for addressing abstract calculus topics (Ardina et al., 2025). Conventional instruction tends to emphasize fixed symbolic methods, which can limit how deeply students make sense of underlying ideas (Niess et al., 2009). In comparison, GeoGebra enables learners to manipulate representations dynamically, creating opportunities for interactive exploration of concepts that are otherwise difficult to grasp without such support (Ziatdinov & Valles Jr., 2022). While many teachers report using GeoGebra for algebraic or geometric topics, far fewer incorporate it routinely into calculus instruction, highlighting the need for professional growth programs that integrate content, pedagogy, and technology (Machromah et al., 2019).

This professional growth module spans three weeks, with each meeting organized as a three-hour working session and drawing upon multiple theoretical and practical frameworks. The module design is informed by the TPACK framework (Mishra & Koehler, 2006), emphasizing the dynamic interaction among disciplinary content, pedagogical strategies, and technological tools, along with Anderson and Shattuck's (2012) guidance on structuring design-based research. Together, these foundations inform an approach designed to support teachers in crafting lessons that use digital tools to clarify integral concepts while addressing common student misconceptions. During hands-on GeoGebra activities, teachers explore tools such as the integral-calculation feature, adjustable parameters, and color-highlighted regions. In some cases, participants are encouraged to experiment with these tools in ways that mirror challenges teachers face when translating abstract calculus concepts into classroom experiences. Through these guided, step-by-step interactions, they learn to create interactive tasks that support deeper reasoning about calculus ideas.

2. LEARNING EFFECTS AND APPLICABLE ELEMENTS

Table 1 outlines the targeted learning effect, effect element, and the corresponding teaching standards. Table 2 summarizes the competence expectations, their associated competence indicators, and the related teaching standards. The standards were published by the International Society for Technology in Education (ISTE, 2017), the Common Core State Standards (2010), and the National Council of Teachers of Mathematics (NCTM, 2014) to guide educators.

Table 1. Learning Effect, Effect Elements, and Teaching Standards

Learning Effect	Effect Element	Teaching Standards
Clearly explain how definite and indefinite integrals differ in both their conceptual interpretation and procedural application.	Explain, using appropriate calculus language, how the two forms of integrals serve different purposes and produce different types of results.	Reasoning-focused learning.
		Application of digital tools in real-world context.
	Explain the significance of the area under a graph when representing accumulated changes in practical contexts.	Supporting the integration of real-world applications into learning.
		Intentional use of digital tools to personalize learning experiences.
Effectively demonstrate how GeoGebra can visually model and support the teaching of integration concepts.	Identify and explain three tools when using GeoGebra to represent integration concepts	Use of digital tools to encourage student agencies.
		Promoting the integration of mathematical representations and tools in mathematics instruction.
	Describe how GeoGebra-generated visual models assist students in making sense of the FTC. $\frac{d}{dx}\left[\int_a^x f(x)dt\right] = f(x)$	Intentional use of instructional design principles to create innovative digital learning environments.
		Provide guidance on advancing conceptual understanding in integration.

Table 2. Competence Expectations, Indicators, and Teaching Standards

Competence Expectations	Competence Indicators	Teaching Standards
Create a lesson plan that integrates GeoGebra in ways that align with relevant mathematics standards and support instruction on definite integrals	Create a lesson plan that uses two of GeoGebra functions to support visual and interactive representations of definite integrals.	Mathematical Representations and Problem-Solving.
		Developing digital tasks that foster deep understanding.
	Show how the lesson's aims correspond with calculus benchmarks and technology-integration expectations.	Implement tasks that foster reasoning and problem-solving.
		Foster and model creativity to communicate ideas through technology.
Deliver a short, simulated lesson that uses GeoGebra to demonstrate the concept of area under a function's graph.	Effectively conduct a 10-minute mock lesson using GeoGebra to model a real-world integration application (e.g., area under a velocity-time graph).	Guide to meaningful mathematical discussions.
		Demonstrate effective use of digital tools to enhance student learning.
	Effectively apply guided questioning and visual prompts to assess how well students are following the lesson.	Apply instructional tasks that support critical thinking and reasoning.
		Offer multiple ways for students to demonstrate mastery of learning.

As shown in Table 1, the first learning effect focuses on helping secondary mathematics teachers use precise calculus terminology to differentiate between indefinite and definite integrals. This effect is grounded in teaching standards that support reasoning-focused learning and the application of digital tools in mathematics teaching to create real-world scenarios (NCTM, 2014). Additionally, they include standards that emphasize the integration of real-world applications and the intentional use of digital tools to personalize learning experiences (ISTE, 2017). The second learning effect centers on building teachers' capacity to choose and apply appropriate GeoGebra tools to illustrate calculus ideas. This involves standards that promote the use of digital tools and the intentional application of instructional design principles to create innovative digital learning environments (ISTE, 2017). Further, they include standards that emphasize the integration of mathematical representations and guide the advancement of conceptual understanding in integration (NCTM, 2014). Together, these standards highlight intentional technology integration and the interpretation of interactive, real-time graphical displays using the Fundamental Theorem of Calculus (FTC). In designing these effects, the study prioritized activities that resemble the difficulties teachers often face when making abstract calculus ideas accessible to students. When considered together, these elements support the development of teachers' content expertise and their capacity to design learner-oriented calculus lessons (Li et al., 2022).

Table 2 summarizes the competence expectations, their associated competence indicators, and the related teaching standards. The first competence expectation centers on teachers' ability to develop calculus-based instructional plans that intentionally integrate GeoGebra features, including the *Integral Between* tool and *dynamic sliders*. In designing these effects, the focus was on tasks that support teachers in weaving together different ways of representing calculus ideas. This is a challenge commonly encountered in professional growth settings. This approach is guided by standards emphasizing mathematical representations and the implementation of digital tasks that require mathematical reasoning (NCTM, 2014). It is also guided by standards supporting the use of digital tasks that promote learner reasoning and creativity (ISTE, 2017). Competence expectation 2 promotes exploratory, discussion-based instruction during practice teaching sessions. In some cases, teachers are invited to experiment with questioning techniques and visual responses while monitoring student thinking. This supports diverse ways for learners to demonstrate understanding. These expectations are based on standards that emphasize meaningful discussion and the development of critical thinking (NCTM, 2014). They also draw on standards that emphasize the effective use of digital tools to encourage learners to demonstrate mastery (ISTE, 2017). Altogether, these effects, informed by the perspectives of Li et al. (2022), are intended to prepare secondary teachers to facilitate rigorous, interactive, and blended calculus learning that is both meaningful and learner-centered.

3. LEARNING EVALUATION

Table 3 outlines the targeted learning effects, their associated elements, and the corresponding evaluation tasks used in both the pre-test and post-test.

Table 3. Learning Effects, Element, and Pre- and Post-Evaluation Tasks

Learning Effect 1:	
Clearly explain how definite integrals and indefinite integrals differ in both their conceptual interpretation and procedural application.	
Effect Element	**Evaluation Tasks**
Element 1: Explain, using appropriate calculus language, how the two forms of integrals serve different purposes and produce different types of results.	**Pre-Evaluation Tasks**
	Given $f(x) = 3x + 2$. 1) Determine the antiderivative $f(x)$. 2) Evaluate $f(x)$ over the interval $1 \leq x \leq 4$. 3) Provide a concise explanation comparing the interpretation and effects of the two integrals using terms such as *antiderivative, area under the curve, numerical value, general solution.*
	Post-Evaluation Tasks
	Let $f(x) = x^2 - 4$. 1) Determine the antiderivative of $f(x)$. 2) Evaluate $f(x)$, over the interval $1 \leq x \leq 2$. 3) Describe the conceptual distinction between the two outcomes using accurate calculus vocabularies such as *antiderivative, area under the curve, numerical value, general solution.*
Element 2: Explain the significance of the area under a graph when representing accumulated changes in practical contexts.	**Pre-Evaluation Tasks**
	A car's speed is defined by $v(t) = 4t$, over the interval $1 \leq t \leq 5$ 1) Set up the integral over the fixed interval that represents the total distance traveled. 2) Evaluate the integrals. 3) Interpret the results.
	Post-Evaluation Tasks
	A car's speed is modeled by $v(t) = 3t + 1$ for $0 \leq t \leq 6$. 1) Set up the integral with limits that represent total area covered. 2) Evaluate the integral. 3) Interpret the meaning of the result in the context of the bicycle's motion.
Learning Effect 2	
Show how GeoGebra's visualization tools can be used to illustrate and teach key ideas related to integration.	
Effect Element	**Evaluation Element**
Element 1: Identify and explain two or more tools when using GeoGebra to represent integration concepts	**Pre-Evaluation Tasks**
	Consider illustrating accumulation with the quadratic function x^2 over the range $[0, a]$. 1) Which GeoGebra tool(s) would you select to represent this visually? 2) Explain how you would configure these tools to help students see how varying the upper integration limit affects the accumulated area.
	Post-Evaluation Tasks
	You are using GeoGebra to represent the integral of $f(x) = \sin(x)$ over the interval from 0 to a. 1) Select two or more GeoGebra features and explain each feature's role in the model. 2) Describe how the features collectively support understanding of accumulated area and the effect of changing integration bounds.
Element 2: Describe how GeoGebra-	**Pre-Evaluation Tasks**
	A student is having difficulty linking integral and derivative processes.

generated visual models assist students in making sense of the FTC.	1) Explain how you may use GeoGebra visualization to clarify this connection. 2) Identify potential areas where learners may struggle while applying GeoGebra features in classrooms.
	Post-Evaluation Tasks
	You are guiding a GeoGebra activity that displays $f(x)$ and its accumulation function $F(x) = \int_a^x f(t)\,dt$ with a variable upper bound. 1) Explain how you would help students recognize that the slope of $F(x)$ corresponds to the value of $f(x)$. 2) Explain the way visualization enhances understanding of integration, accumulation, differentiation, and rates of change.

To assess Learning Effect 1, teachers' ability to accurately define and explain how indefinite integrals differ from definite integrals, both pre-treatment and post-treatment evaluation tasks are carefully structured to capture how teachers progress from solving simple mathematical tasks to addressing more complex tasks that require deeper understanding. In the pre-evaluation phase, teachers are given a straightforward linear function, $f(x) = 3x + 2$, and asked to find both the antiderivative and the definite integral evaluated over the interval $[0,4]$. They are then prompted to provide a brief written explanation clarifying the conceptual differences between the two results. For the post-evaluation, the task mirrors the pre-evaluation in structure but introduces a more complex function, $\int f(x) = x^2 - 4$. In addition to calculating the results, teachers are asked to demonstrate the use of accurate calculus terminology, including concepts such as numerical value, area under the curve, and antiderivative. The progression was designed with reference to Sarama et al.'s (2016) work, highlighting how learning-trajectory–based treatments can influence teachers' instructional practices. Building on this foundation, maintaining a parallel evaluation structure allows for direct comparisons over time and, in some cases, highlights a shift from surface-level procedural skills to more robust conceptual understanding, as described by Anderson et al. (2012). To support this continuity, these evaluations are integrated into regular instruction at the start of Week 1 and again at the conclusion of Week 3, enabling deliberate data collection without interrupting the flow of teaching.

To assess the first aspect of Learning Effect 2, teachers are required to identify any two features of GeoGebra and explain how they can be used to represent key calculus concepts. In the pre-evaluation, for example, teachers are given the integral of $f(x) = x^2$ and asked to select GeoGebra features that are most suitable for illustrating cumulative change over the interval $[0, a]$, as well as how these tools might be configured to demonstrate the effect of a changing upper bound. In designing this task, the intention was to capture not only teachers' procedural decisions but also their initial confidence and familiarity with dynamic visual representations. For the post-evaluation, the mathematical context shifts to $f(x)\,dx = sin(x)$, increasing the cognitive complexity while maintaining the same overall task structure. Teachers are again asked to describe how to use the GeoGebra tools introduced earlier; in some cases, they are also prompted to explain how these features interact to help students understand concepts such as the area under a curve and variable limits of integration. Across both evaluations, the focus is on capturing not only procedural skills but also technological and pedagogical reasoning, with the post-evaluation placing greater emphasis on coherent instructional planning. These tasks are integrated into instructional lessons that utilize GeoGebra. The pre-evaluation is conducted during the initial modeling session and requires approximately *seven* to *ten* minutes, whereas the post-evaluation is embedded within a summative instructional task. This approach to evaluation integration sustains instructional continuity and yields ongoing, informative feedback on participants' developing understanding, reflecting established recommendations for embedding evaluation within teaching (Wiliam, 2011; Neely et al., 2000).

To evaluate the second facet of Learning Effect 2, how GeoGebra graphical displays can support conceptual comprehension of the FTC, teachers complete tasks designed to emphasize conceptual reasoning. For example, during the pre-test, teachers are required to describe the graph of $\int_a^x f(x)$ dt in a way that enables a peer to distinguish between integration and differentiation, and to predict possible hurdles that learners are likely to encounter when working with graphical tools. During the post-test, teachers illustrate $f(x)$ and its integral function, $F(x) = \int_a^x f(t)\,dt$, by varying the limits of integration (Finney et al., 2007). In some situations, teachers are invited to discuss strategies for supporting students in recognizing the relationship between the slope of $F(x)$ and the value of $f(x)$, using interactive visual representations to reinforce the FTC (Larson et al., 2002; Finney et al., 2007). This approach evaluates not only conceptual understanding but also pedagogical reasoning, with the post-evaluation placing greater demands on instructional planning. In designing these tasks, the intention was to reflect real classroom challenges and support teachers' ability to integrate dynamic representations effectively. Consistent with Yi et al. (2020), the use of visualizations encourages deeper conceptual comprehension.

3.1. Instructional Competencies Evaluation

Table 4 describes the instructional competencies, their corresponding indicators, and the rubric used to evaluate pre-test and post-test tasks.

Table 4. Instructional Competencies, Corresponding Indicators, and Rubrics

<table>
<tr><th colspan="6">Instructional Competencies 1:</th></tr>
<tr><td colspan="6">Create a lesson plan that integrates GeoGebra in ways that align with relevant mathematics standards and support instruction on definite integrals</td></tr>
<tr><th>Effect Element</th><th colspan="5">Evaluation Measures</th></tr>
<tr><td rowspan="7">Element 1:
Create a lesson plan that uses two of GeoGebra functions to support visual and interactive representations of definite integrals.</td><td colspan="5">Pre-Evaluation Tasks
Given $f(x) = 3x + 2$.
1) Determine the antiderivative $f(x)$.
2) Evaluate the value of $f(x)$ from 1 to 4.
3) Explain how indefinite and definite integrals differ in interpretation and outcome terms such as antiderivative, area under the curve, numerical value, general solution.</td></tr>
<tr><th>Standard</th><th>4 - Satisfactory</th><th>3 - Mastery</th><th>2 - Good</th><th>1 - Weak</th></tr>
<tr><td>Application of GeoGebra Tools</td><td>Effectively combines two or more GeoGebra tools (e.g., Integral Between, sliders) with clear instructional intent and mathematical precision.</td><td>Employs at least two tools with mostly accurate use, though the instructional purpose is not clearly articulated.</td><td>Relies on a single tool or applies multiple tools with limited effectiveness and minor errors.</td><td>Tools are absent, misapplied, or not aligned with the instructional goals.</td></tr>
<tr><td>Depiction of Integration Concepts</td><td>Visual representations are explicitly linked to definite integrals, promoting strong conceptual understanding.</td><td>Visual representations support integration concepts but show limited or unclear connections to definite integrals.</td><td>Concepts are addressed at a surface level, with visuals offering minimal mathematical support.</td><td>Visual tools are absent or fail to convey integration concepts.</td></tr>
<tr><td colspan="5">Post-Evaluation Tasks
Let $f(x) = x^2 - 4$.
1) Compute $\int f(x)\,dx$.
2) Compute $\int_1^2 f(x)\,dx$.
3) Explain the conceptual difference between the two results using precise calculus terms such as antiderivative, area under the curve, numerical value, general solution.</td></tr>
<tr><td>Application of GeoGebra Tools</td><td>Seamlessly integrates multiple relevant tools with clear mathematical purpose and instructional value.</td><td>Uses two tools correctly with reasonable effectiveness, though some aspects may require improvement.</td><td>Uses one or two tools inconsistently, with limited clarity in instructional intent.</td><td>GeoGebra tools are missing or misused, leading to inaccurate representation of integration concepts.</td></tr>
<tr><td>Depiction of Integration Concepts</td><td>Visualizations accurately model definite integrals, showing setup, graphical output, and links to the FTC.</td><td>Visuals aid understanding of definite integrals, generally linking well to core concepts.</td><td>Representations show partial connection to integration concepts but may be unclear or contain errors.</td><td>Visuals fail to represent integration concepts or do so incorrectly.</td></tr>
<tr><td>Element 2:
Show how the lesson's aims correspond with calculus</td><td colspan="5">Pre-Evaluation Tasks
A car's speed is defined by $v(t) = 4t$, over the interval $1 \le t \le 5$
1) Set up the integral over the fixed interval that represents the total distance traveled.
2) Evaluate the integrals.
3) Interpret the results.</td></tr>
</table>

benchmarks and technology-integration expectations.	Connection to Calculus Content Standards	Lesson learning targets clearly align with key secondary school calculus standards, including definite integrals and the FTC.	Goals are coherent with most relevant calculus standards, though some elements may be incomplete.	Objectives address basic calculus concepts but show limited alignment with specific standards.	Objectives are disconnected from, or only weakly related to, established calculus standards.
	Alignment with Technology Integration Standards	Objectives align with ISTE standards, fostering student creativity and problem-solving through technology (e.g., GeoGebra).	References ISTE standards, but links to creativity and problem-solving are vague or implied.	Technology is used, but connections to ISTE standards and creative learning are unclear or absent.	Shows minimal or no alignment with ISTE technology standards.
	Post-Evaluation Tasks A car's speed is modeled by $v(t) = 3t + 1$ for $0 \leq t \leq 6$. 1) Set up the integral with limits that represent the total area covered. 2) Evaluate the integral. 3) Interpret the meaning of the result in the context of the car's motion.				
	Connection to Calculus Content Standards	Instructional goals match secondary calculus standards while supporting reasoning and problem-solving with bounded integrals.	Goals are well aligned with calculus standards, indicating purposeful instruction.	Alignment is apparent, but its connection to reasoning and problem-solving remains unclear.	Objectives are vague, partial, or not aligned with calculus content.
	Alignment with Technology Integration Standards	Technology integration goals align with ISTE Standard 2.6, fostering creative expression using GeoGebra.	Objectives cite ISTE 2.6, supporting student technology use with limited creative engagement.	Technology use is noted broadly, but links to creative expression and ISTE standards are weak.	Technology objectives are absent or fail to align with ISTE standards.
Instructional Competencies 2					
Deliver a short, simulated lesson that uses GeoGebra to demonstrate the concept of area under a function's graph.					
Effect Element	**Evaluation Element**				
Element 1: Effectively conduct a 10-minute mock lesson using GeoGebra to model a real-world integration application (e.g., area under a velocity-time graph).	**Pre-Evaluation Tasks** Consider illustrating accumulation with the quadratic function x^2 over the range [0, a]. 1) Which GeoGebra tool(s) would you select to represent this visually? 2) Explain how you would configure these tools to help students see how varying the upper integration limit affects the accumulated area.				
	Standard	**4 - Satisfactory**	**3 - Mastery**	**2 - Good**	**1 - Weak**
	Precision in mathematical representations	Applied contexts are represented precisely through appropriate integration techniques and calculus language.	The solution is largely correct, with small mathematical or modeling inaccuracies.	The solution is included but shows major mathematical mistakes or unclear reasoning.	The solution is incorrect or not connected to integration ideas.
	Effective Use of GeoGebra Tools	Integration ideas are clearly illustrated through effective use of GeoGebra tools.	Tools are used appropriately, but their full potential to enhance student understanding is not fully realized.	Tools are often underutilized or applied in ways that hinder clarity.	Little to no use of GeoGebra or other tools misrepresents integration concepts.
	Post-Evaluation Tasks You are using GeoGebra to represent the integral of $f(x) = \sin(x)$ over the interval from 0 to a. 1) Select two or more GeoGebra features and explain each feature's role in the model. 2) Describe how the features collectively support understanding of accumulated area and the effect of changing integration bounds.				
	Precision in Mathematical Modeling	The applied integration tasks are represented accurately and clearly, facilitating	The representation is largely correct and understandable, with slight imperfections, providing moderate support for inquiry.	The model contains clear errors or insufficient explanations, restricting	The model is inaccurate or confusing, providing no support for student inquiry.

		student inquiry and exploration.		opportunities for student inquiry.	
	Proficient Application of GeoGebra Tools	Skillfully integrates two or more GeoGebra tools to clearly and dynamically represent integration concepts.	Effectively applies two or more GeoGebra tools, with some room to enhance dynamic interaction.	Applies tools inconsistently or superficially, resulting in unclear visualizations.	GeoGebra tools are absent or misapplied, providing no meaningful visual representation of integration.
Element 2: Effectively apply guided questioning and visual prompts to assess how well students are following the lesson.	**Pre-Evaluation Tasks** A student is having difficulty linking integral and derivative processes. 1) Explain how you may use GeoGebra visualization to clarify this connection. 2) Identify potential areas where learners may struggle while applying GeoGebra features in classrooms.				
	Techniques for Eliciting Student Thinking	Employs thoughtful, open-ended questions to uncover student reasoning, understanding, and potential misconceptions.	Poses appropriate questions to elicit student responses, though follow-up and depth are limited.	Relies primarily on closed or factual questions, offering little opportunity for student engagement.	Seldom or poorly employs questions to gauge student comprehension.
	Providing Visual Feedback with GeoGebra	Effectively employs visual tools (e.g., shaded regions, sliders) to respond meaningfully to student contributions.	Uses visual aids to enhance understanding, though connections to student input are partial or unclear.	Employ GeoGebra visuals without actively addressing student responses or questions.	Fails to employ visual tools, or their use is unrelated to student comprehension.
	Post-Evaluation Tasks You are guiding a GeoGebra activity that displays $f(x)$ and its accumulation function $F(x) = \int_a^x f(t)\,dt$ with a variable upper bound. 1) Explain how you would help students recognize that the slope of $F(x)$ corresponds to the value of $f(x)$. 2) Explain the way visualization enhances understanding of integration, accumulation, differentiation, and rates of change.				
	Techniques for Eliciting Student Thinking	Deliberately uses multi-level, open-ended questions to promote inquiry and reveal student thinking.	Uses a variety of questions to assess comprehension and prompt student explanations.	Uses questioning sparingly, with minimal adaptation to student responses.	Rarely asks questions, offering little opportunity to explore student understanding.
	Providing Visual Feedback with GeoGebra	Actively employs GeoGebra to illustrate concepts, responding to student input and adjusting instruction accordingly.	Offers visual cues that directly connect to teaching points and student contributions.	Uses visuals mainly for demonstration, with minimal responsiveness or adjustment.	Does not employ GeoGebra effectively to enhance student understanding or respond to feedback.

Scoring Rubric: 7 and above – Satisfactory; 5 to 6 marks – Mastery; 3 to 4 marks – Good; 0 to 2 marks – Weak

To assess the first element of Instructional Competencies 1, creating an instructional plan that integrates a minimum of two GeoGebra features to graphically depict integration, pre- and post-evaluation rubrics are structured around two core criteria: purposeful use of digital tools and precision in conceptual depiction (Hohenwarter & Hohenwarter, 2002). During the initial evaluation, Criterion 1 analyzes teachers' use of features such as *adjustable sliders* and *Integral Between*, with top ratings assigned when the implementation is intentional and mathematically accurate. The second criterion evaluates how well graphical representations correctly portray definite integrals and promote conceptual understanding. In the follow-up evaluation, the same scoring structure is retained, although competence expectations are raised. The first criterion now prioritizes indications that the chosen features improve teaching effectiveness, whereas the second criterion concentrates on clearly linking graphical representations to the FTC. Both scoring guides maintain a four-tier rating system, spanning from weak to satisfactory, to ensure consistency and monitor improvement in instructional design quality (Finney et al., 2007).

To assess the first element of Instructional Competencies 2, conducting a brief simulated lesson with GeoGebra to represent an applied integration scenario, pre- and post-evaluation rubrics are structured around two central dimensions: mathematical representation and purposeful technology use. The first criterion evaluates the correctness and authenticity of modeling applied scenarios, such as determining distance from a velocity–time graph, awarding top ratings when instructors use accurate calculus language and well-justified contextual reasoning. The second criterion concentrates on combining GeoGebra functions, with strong competence showing clear instructional value and improved explanation through features such as adjustable *sliders* and highlighted areas. In the follow-up evaluation, competence standards are raised. The first criterion now prioritizes inquiry-oriented facilitation, while the second criterion stresses interactive, learner-focused engagement with technological resources. For instance, an instructional activity that represents $v(t) = 4t$ by using adjustable sliders to vary the time range, while supporting learners in reasoning about accumulated distance and changing rates, would achieve the top competence rating. These scoring guides were created to keep category structure stable while gradually increasing competence demands, enabling equitable, development-focused evaluation. Studies indicate that this form of structured support strengthens instructional design, supports teachers' adoption of quality benchmarks, and improves teaching proficiency (Neely et al., 2000; Yi et al., 2020).

Initial and follow-up evaluations use shared scoring dimensions to assess key skills in mathematical correctness and purposeful technology use, while progressively stressing instructional depth and learner involvement. By maintaining stable criteria while increasing post-evaluation standards, this structure allows dependable tracking of teacher development and encourages the gradual advancement of high-impact, blended instructional practices. The scoring guides are embedded within routine instructional tasks to preserve instructional continuity (Wiliam, 2011). For Instructional Competencies 1, the initial evaluation occurs after a GeoGebra walkthrough and requires participants to create a preliminary instructional plan, lasting approximately *fifteen to twenty* minutes. The follow-up evaluation extends the initial version by integrating peer input and reflective feedback to generate a revised instructional plan. For Instructional Competencies 2, the baseline evaluation follows an initial simulated lesson demonstrating a simple applied scenario, while the follow-up evaluation occurs after a second simulated lesson involving more advanced modeling, dynamic real-time visuals, and increased learner interaction. Integrating evaluations in this manner supports meaningful learning, aligning with the recommendations of Wiliam (2011) and Neely et al. (2000). In addition to measurement, these scoring guides function as resources for feedback and professional growth. They make competence criteria explicit, encourage reflective teaching, and uphold ISTE (2017) and NCTM (2014) guidelines, promoting purposeful technology integration in calculus instruction.

4. TREATMENT AND DATA COLLECTION

Table 5 summarizes the implementation and information gathering plan for the study, specifying activity sequencing, targeted results, sources, and duration per implementation.

Table 5. Implementation and Information Gathering

Weeks	Implementations in Sequence	Targeted Effect	Information Gathered	Time Needed
WK (i)	**Implementation one**: Instructor-guided presentation ***Emphasis***: Understanding the distinctions in underlying ideas and procedures among integrals with and without limits (PCK). ***Task***: Instructor-led lecture, structured note-taking, and worked examples	**Learning Effect 1**: Teachers are expected to clearly show comprehension of both the underlying ideas and procedures distinctions among integrals with and without limits	**Data:** Pre-evaluation answers to conceptual and computational questions, along with students' self-reflections.	59 minutes
		Element 1: Teachers to clearly articulate the differences between definite and indefinite integrals using correct calculus terminology.		
		Element 2: Teachers should correctly interpret the area under a curve in real-world contexts and explain its calculation through integration.		
		Learning Effect 1: Teachers should exhibit clear mastery of underlying concepts and	**Data:**	

Weeks	Implementations in Sequence	Targeted Effect	Information Gathered	Time Needed
	Implementation two: Collective reasoning and problem resolution ***Emphasis***: Applying calculus applications in authentic contexts (PCK) ***Task***: Solve motion-related problems in small groups and present solutions.	procedures for integrals with and without limits. **Element 1:** Teachers should clearly explain the distinction between bounded and unbounded integrals using accurate calculus language. **Element 2:** Teachers should accurately interpret area under a curve in applied contexts and explain how integration determines it.	Collaborative solution outputs and instructor observation records	59 minutes
	Implementation three: Recorded instruction reviews and reflection. ***Emphasis***: Understanding area as accumulation and differentiating integral forms. ***Task***: View digital display illustrating applied integration and engage in a structured discussion.	**Learning Effect 1**: Teachers should effectively demonstrate comprehension of both the key ideas and procedures distinguishing bounded and unbounded integrals. **Element 1:** Teachers clearly describe, using correct calculus language, how definite integrals differ from indefinite integrals and what each represents. **Element 2**: Teachers should correctly interpret accumulated area beneath a graph in applied settings contexts.	**Data:** Discussion records and exit slips	49 minutes
WK (ii)	**Implementation one:** GeoGebra Introduction and Demonstration ***Emphasis***: Essential GeoGebra tools for integration (TCK) ***Task***: Hands-on use of sliders, shaded areas, and integral tools	**Learning Effect 2:** Teachers should effectively show how GeoGebra can visually model and support the teaching of integration concepts. **Element 1:** Teachers should correctly describe and apply at least three GeoGebra features to model calculus ideas. **Element 2:** Teachers are expected to explain how GeoGebra representations improve student comprehension of the FTC.	**Data:** Pre-evaluation using tool-description tasks	59 minutes
	Implementation Two: GeoGebra-based task exploring area accumulation for a curve ***Emphasis:*** Representing motion and accumulation with interactive sliders and area displays (TCK) ***Task***: Use GeoGebra to represent $\upsilon(t) = 4t$, vary the interval with a slider, and interpret outcomes.	**Learning Effect 2:** Teachers should effectively demonstrate how GeoGebra can visually model and facilitate the teaching of integration concepts. **Element 1**: Teachers should correctly describe and apply at least two GeoGebra features to model calculus ideas. **Element 2:** Teachers are expected to explain how GeoGebra representations enhance comprehension of the FTC.	**Data:** Student screenshots and summaries of peer feedback	59 minutes
	Implementation Three: Facilitated dialogue on GeoGebra and the FTC ***Emphasis:*** Relating graphical displays to the FTC (TPCK) ***Task***: Study the function $F(x) = \int_a^x f(t)\,dt$ hence explain its connection to the FTC.	**Learning Effect 2**: Teachers should effectively show how GeoGebra can visually model and support instruction of integration concepts. **Element 1**: Teachers should correctly describe and apply two or more GeoGebra features to model calculus ideas. **Element 2**: Teachers are expected to explain how GeoGebra representations enhance students' understanding of FTC.	**Data:** Instructor notes, exit slips, and concept maps	49 minutes
WK (iii)	**Implementation One**: Teaching plan development workshop ***Emphasis***: Developing instruction that integrates GeoGebra features (PK) ***Task***: Review sample lessons, collaboratively plan in pairs with instructor input	**Demonstrated Competency 1:** Teachers should effectively develop a standards-aligned instructional plan integrating GeoGebra for definite integrals. **Element 1:** Teachers effectively construct an instructional plan that employs multiple GeoGebra features to generate interactive representations of definite integrals **Element 2**: Teachers should ensure lesson objectives are properly aligned with secondary school calculus content and technology integration standards.	**Data:** Draft lesson plans and peer and instructor evaluation rubrics	51 minutes
	Implementation Two: Simulated instruction session	**Demonstrated Competency 2**: Effectively lead a practice lesson employing GeoGebra focused on curve area and inquiry	**Data:**	59 minutes

Weeks	Implementations in Sequence	Targeted Effect	Information Gathered	Time Needed
	Emphasis: Practicing calculus instruction supported by GeoGebra. ***Task***: Deliver a simulated lesson to peers and receive feedback.	**Element 1:** Teachers should conduct a short instructional rehearsal with GeoGebra to illustrate an applied integration scenario. **Element 2:** Teachers should skillfully use questions and visual indicators to evaluate comprehension throughout the lesson.	Lesson rubrics and video recordings	
	Implementation Three: End-of-module evaluation and reflection ***Emphasis***: Assess learning progress (professional identity) ***Task***: Complete post-evaluation tasks and compose personal reflection.	**Learning Effect + Demonstrated Competency** **Element 1**: Show conceptual comprehension **Element 2**: Assess GeoGebra's influence	**Data:** Post-evaluation tasks: reflective essays	51 minutes

4.1. Model Treatment

The lesson design draws on the TPACK model to connect subject matter, teaching practice, and technological proficiency in digital learning contexts (Mishra & Koehler, 2006). The first week focuses on establishing core conceptual understanding of integration. Treatment 1 combines lecture and guided notes to clarify how indefinite integrals represent families of antiderivatives, whereas definite integrals yield a single accumulated value, highlighting CK through key calculus concepts such as limits, accumulation, and signed area. By pairing sample problems with guided notes, the intent is to strengthen procedural fluency while supporting clear mathematical reasoning (Mishra & Koehler, 2006; Niess et al., 2009). Treatment 2 engages teachers in group discussions and problem-solving tasks grounded in real-world motion scenarios. Some tasks support teachers in anticipating learner responses and facilitating concept-driven discussions, thereby fostering PCK development (Mishra & Koehler, 2006). The final treatment relies on video reflection to connect integral concepts to practical motion-based contexts. Reflective guidance strengthens self-awareness and critical examination of pedagogical decisions and learner perspectives (Niess et al., 2009).

Week two centers on incorporating digital tools into calculus instruction. The initial activity introduces GeoGebra through demonstrations, supporting TCK as teachers engage with dynamic integral features. Hands-on engagement with evolving integral representations prepares participants for more complex digital construction activities. In Treatment 2, teachers engage in a GeoGebra activity that models the area under a curve. Exploration of interval variation and resulting area outcomes strengthens teachers' understanding of integrals as net change through digital representations (Hohenwarter et al., 2015; Ziatdinov & Valles Jr., 2022). Treatment 3 features a facilitated discussion on GeoGebra and the FTC, with the goal of developing full TPACK (Mishra & Koehler, 2006) by guiding teachers in interpreting the FTC using interactive, real-time graphical displays. According to Niess et al. (2009), such collaborative dialogue supports connections between interactive models and formal calculus reasoning, enhancing teachers' ability to communicate abstract ideas through digital tools.

Week 3 emphasizes applying conceptual and technological knowledge in practical classroom contexts. Treatment 1 consists of a lesson plan writing workshop in which teachers co-design lessons incorporating GeoGebra, targeting PK and TPACK (Mishra & Koehler, 2006). Using sample templates and peer collaboration, participants practice applying subject knowledge and digital skills within an organized instructional planning structure. The second treatment involves a practice-based teaching activity with presentations, providing teachers opportunities to rehearse lessons and receive structured feedback, thereby improving instructional fluency and the effective integration of GeoGebra in classroom practice, as guided by Niess et al. (2009). The final instructional phase links outcome-based evaluation with reflection to document progress in disciplinary knowledge and pedagogical competence. Reflective prompts support goal setting, independent learning, and professional identity development. Finally, Week 3 consolidates instructional elements using a design-based framework centered on refinement, authenticity, and theoretical alignment (Anderson & Shattuck, 2012; Seidel et al., 2007).

4.2. Information Gathering

Across the module, teachers' progress is examined through ongoing and cumulative evaluations matched to weekly emphases on concepts, technology use, and instruction. The first week includes an early diagnostic of integration understanding and reflective self-appraisals of confidence with definite and indefinite integrals. These reflections encourage metacognition, prompting teachers to articulate their understanding and identify gaps. During Treatment 2, small-group problem-solving activities produce artifacts that are reviewed alongside instructor notes, providing insight into collaborative reasoning and the application of calculus to real-world scenarios. Treatment 3 combines video-based analysis with guided discussions, exit tickets, and discussion notes to assess conceptual interpretation, problem-solving strategies, and negotiation of ideas. Evaluation design emphasizes procedural competence and conceptual insight to support TPACK-informed instructional decision making (Mishra & Koehler, 2006).

Week two centers on strengthening GeoGebra skills through diagnostics, reflective documentation, and the creation of interactive digital models. The resulting artifacts reveal accuracy, collaborative effort, and the ways in which technology supports conceptual understanding and mathematical reasoning in calculus. Treatment 3 involves a guided exploration of the FTC using GeoGebra. Instructor observations are recorded, and teachers submit exit slips and concept maps to capture snapshots of conceptual understanding and cognitive connections. By integrating observation, artifacts, and peer interaction within authentic tasks, these evaluations provide low-disruption yet meaningful measures of technological and representational competence, aligned with TPACK (Mishra & Koehler, 2006) and ISTE (2017).

Week three examines instructional competence through collaborative lesson design using GeoGebra, assessed for coherence, precision, and technology application. Peer and facilitator feedback guides revisions. Teachers then complete recorded teaching rehearsals, assessed for instructional clarity and effective digital tool use, strengthening fluency and reflective practice. The final treatment assesses conceptual gains and includes reflective analysis of instructional decisions and outcomes. Evaluation is embedded within instructions to avoid disrupting learning flow. The first *ten* to *fifteen* minutes of Weeks 1 and 2 are used for diagnostics, while collaborative sessions generate group artifacts and discussion documentation. Task artifacts and peer responses are submitted digitally upon completion, with instructor observations captured using a formal checklist. Week three also includes workshop-based review of lesson drafts and peer rubrics, concluding with capstone evaluations. Recorded teaching rehearsals and digitally submitted brief evaluations conclude the class sessions. Consistent with Wiliam (2011) and Neely et al. (2000), this approach integrates formative and summative evaluation into instruction, minimizes disruption, and supports continuous, goal-aligned data collection.

5. Study Design and Data Application

The following section outlines how information, including embedded evaluations, observations, and reflective artifacts, from the professional growth module is applied to examine secondary teachers' development in calculus knowledge, technology use, and instructional practice.

5.1. Research Emphasis

This investigation analyzes how a blended DBR professional growth model improves teachers' use of GeoGebra in integral calculus instruction. In Week 1, the focus is on deepening conceptual understanding of definite and indefinite integrals. Learning is assessed through a combination of pre-evaluations, reflective writing, collaborative artifacts, and guided discussions, which together provide insight into both procedural skill and conceptual reasoning. Week 2 emphasizes developing technological fluency and representational competence with GeoGebra. Evaluation in this phase includes tool-based pre-evaluations, instructor observations, and modeling tasks, capturing how teachers manipulate digital tools to convey integral concepts. In Week 3, the emphasis shifts toward cultivating instructional competence, measured through standards-aligned lesson plans, video-recorded mock teaching sessions, and reflective analyses of teaching practices. Following Joseph's (2004) design-based approach, the module allows iterative refinement in authentic instructional settings, supporting the progressive development of teachers'

TPACK. In designing these evaluations, the priority was to select tasks that mirror real classroom challenges, providing both actionable feedback and evidence of growth.

5.2. Research Questions

a. In what ways does participation in Week 1 of the professional growth program support secondary mathematics teachers' development of an abstract understanding of bounded and unbounded integrals through guided instruction, collaborative problem-solving, and structured reflection?
b. In what ways do Week 2 GeoGebra tasks support secondary teachers' growth in computational proficiency and visual understanding in calculus?
c. In what ways do Week 3 activities enable secondary teachers to show instructional skill in using GeoGebra for calculus through lesson design, simulated teaching, and reflective review?

5.3. Application of Research Results to Module Improvement

Examining teachers' development across content, digital, and teaching areas offers valuable insight for refining the professional growth module. First, monitoring shifts in conceptual understanding through reflective tasks and group artifacts helps identify where Week 1 strategies may require adjustment. In some cases, if teachers demonstrate fragmented or surface-level reasoning, additional scaffolding, visual support, or peer modeling can be introduced to strengthen comprehension. Second, evaluating technological fluency during Week 2 through GeoGebra tasks allows for fine-tuning tool introductions, guided explorations, and support resources. For example, observed challenges or misalignment with learning goals may suggest modifications to instructional timing, task structure, or peer feedback to increase both intellectual challenge and authenticity. Third, examining GeoGebra integration across planning, teaching practice, and reflection shapes the pedagogical priorities of the professional growth program. If teaching rehearsals reveal limited use of dynamic visuals, greater emphasis can be placed on representational instruction and learner-centered approaches. Consistent with Neely et al. (2000), triangulating data from pre- and post-evaluations, observations, and reflective artifacts enables the module to generate detailed feedback that guides coherence among learning goals, evaluation, and instructional tasks. In designing this investigation, the intention was to maintain evaluations as embedded, low-disruption elements closely connected to authentic classroom practice, thereby supporting iterative refinement of both content and pedagogical strategies (Zheng, 2015).

5.4. Advancement of Knowledge in Mathematics

This work extends scholarship on blended learning, DBR, and TPACK by using structured evaluations that link content comprehension, use of digital tools, and instructional practice in professional growth. In some cases, these evaluations create space for teacher self-examination and reasoning while experimenting with interactive tools, supporting both representational competence and metacognition (Mishra & Koehler, 2006) and aligning with ISTE (2017) and NCTM (2014) standards. Ardina et al. (2025) observed higher levels of student interest and participation, along with clearer comprehension of targeted concepts, when GeoGebra is blended into mathematics instruction. Furthermore, professional growth that addresses teachers' pedagogical beliefs, attitudes, and competencies has been shown to positively influence technology dispositions, underscoring the importance of intentional instructional design (Benning et al., 2023). This study contributes to DBR literature by offering an iterative, field-tested model that triangulates reflections, competence artifacts, and observational data (Joseph, 2004; Zheng, 2015). In designing the module, the intention was to move beyond demonstration-based approaches, which have been criticized for limiting teacher engagement in pedagogical decision making (Mensah, 2025) and instead promote reflective and participatory practices that facilitate the collaborative development of techno-didactic knowledge. GeoGebra further supports comprehension of abstract calculus concepts through dynamic visualization and visual reasoning (Hohenwarter & Hohenwarter, 2008). Research indicates that interactive visual tools are effective across mathematics domains, particularly in geometry and calculus, in approximately 80% of studies reviewed (Uwurukundo et al., 2020). Additionally, video-based professional growth has been shown to strengthen teachers' declarative knowledge and their ability to implement GeoGebra-supported visualizations effectively (Zhang, 2025). By embedding formative and summative evaluations within collaborative problem-solving, modeling, and lesson-planning activities, this study fosters self-regulated learning and enhances teaching proficiency. In practice, this design addresses common professional growth challenges, including sustained support, equitable access to technology, and infrastructure constraints (Benning et al., 2023), while encouraging teachers to actively integrate technology into meaningful calculus instruction.

5.5. Mapping Research Questions to Evidence and Data

Table 6 below shows the alignment between the data to be collected from the module, the research questions, and the information necessary for answering each of the research questions.

Table 6. Alignment of Research Questions, Information for Questions, and Data Sources

	Information	Data Instrument	Data Source
1.	Indicators of conceptual growth through pre/post evaluations, reflections, and collaborative artifacts	Observation	Tracking shifts in instructional practices throughout module sessions
		Documents	Collective task evidence
		Surveys	Free-response survey questions for module refinement
		Assessment	Before-and-after evaluations of integral understanding
2.	Quality of reasoning in group activities and instructor observations	Observation	a) Group-based activities b) Facilitator observations
3.	Expressed understanding in reflective essays and journals	Documents	Written reflections
		Surveys	Free-response survey questions for module refinement
4.	Skill in using GeoGebra tools, demonstrated through tasks and diagnostic checks	Observation	Lesson delivery emphasizing GeoGebra integration
		Documents	GeoGebra-generated outputs
		Assessment	GeoGebra-supported lesson planning task
5.	Accuracy and impact of interactive models and collaborative feedback	Documents	a) Screen captures b) Feedback
6.	Documented conceptual insights from exit slips and mapping activities	Documents	Conceptual diagrams
		Surveys	Free-response survey questions for module refinement
		Assessments	Before-and-after evaluations of integral understanding
7.	Clarity and consistency in blended learning lesson plans	Documents	Lesson designs
8.	Effectiveness of simulated teaching, including technology integration	Observations	Videos
		Assessments	Evaluation of reflective work and teaching materials using rubrics
9.	Richness of reflective insights on teaching practice and learning progress	Observation	Written reflections

As shown in Table 6, to address RQ1, *in what ways does participation in Week 1 of the professional growth program support secondary mathematics teachers' development of abstract understanding of bounded and unbounded integrals through guided instruction, collaborative problem-solving, and structured reflection?* Information 1, 2, and 3 act as a measure for selecting the required research instrument for collecting data. As observed by Saraha et al. (2016), conceptual development is examined through parallel pre- and post-evaluation tasks. These tasks allow changes in reasoning, language use, and interpretation to be traced across the module, providing both procedural and conceptual evidence. In this context, recorded observations and instructor notes from collaborative tasks may reveal patterns of reasoning, interaction, and learner engagement. In line with Joseph (2004), group-created materials demonstrate strategy growth and meaning construction. Reflections reveal teachers' metacognitive processing of calculus concepts.

To address RQ2, *in what ways do Week 2 GeoGebra tasks support secondary teachers' growth in computational proficiency and visual understanding in calculus?* Information 4, 5, and 6 act as a measure for selecting the required research instrument for collecting data. In this context, real-time monitoring during GeoGebra activities may be used to capture information on how teachers manipulate dynamic features and interpret visualizations. Additionally, GeoGebra file records and visual captures document representational accuracy and conceptual coherence. Collaborative feedback may also be considered to offer insight into the communicative strength and clarity of participants' digital representations. Complementary data from concept maps, exit slips, and survey responses may be used to illustrate how teachers connect visual and symbolic ideas. In some cases, these data reveal teachers' perceptions of developing technological fluency (Niess et al., 2009).

To address RQ3, *in what ways do Week 3 activities enable secondary teachers to show instructional skill in using GeoGebra for calculus through lesson design, simulated teaching, and reflective review?* Information 7, 8, and 9 act as a measure for selecting the required research instruments for collecting data. In this context, lesson plans are analyzed using analytic rubrics to evaluate coherence, conceptual rigor, and the integration of GeoGebra into pedagogy (Niess et al., 2009). Consistent with Seidel and Shavelson (2007), recorded simulated lessons, combined with observation records, may be used to capture how teachers apply instructional technologies while facilitating learning conversations. Additionally, competence is measured through rubrics, while reflections and surveys document teaching rationale and plans for future GeoGebra implementation.

5.6. Limitations of the Module

The limitations of this study reflect ongoing challenges in GeoGebra integration research. The three-week professional growth module emphasizes the need for continuous support and sustained professional development, as emphasized by Benning et al. (2023). Additionally, Ardina et al. (2025) note that structured pedagogical support and systematic curriculum integration are critical for successful implementation, suggesting that short-term treatments may be insufficient. While self-reported data and instructional artifacts provide valuable insight into reflective processes, they also reflect the methodological constraints identified by Zhang (2025), who stressed the importance of combining observational data with self-reports for a more comprehensive examination of teaching practice. Moreover, the study's focus on a small sample of secondary mathematics teachers mirrors limitations noted by Mensah (2025) and Benning et al. (2023), highlighting how restricted sample sizes can limit generalizability across educational contexts. Future research should address gaps identified in the literature, including variations in GeoGebra's effectiveness across mathematical domains and instructional approaches (Uwurukundo et al., 2020), as well as the extended outcomes of technology adoption for diverse learners and Evaluation design (Ardina et al., 2025). Contextual factors such as administrative support, infrastructure, and teacher agency also warrant further investigation (Benning et al., 2023; Mensah, 2025). Comparative studies examining multiple technological tools could extend understanding beyond GeoGebra, addressing Uwurukundo et al.'s (2020) observation that positive outcomes depend on integration quality and revealing patterns of effective blended mathematics instruction across tools and contexts.

6. CONCLUSION

This study illustrates how a DBR approach can effectively guide the design and refinement of a professional growth module aimed at supporting secondary school mathematics teachers in teaching definite and indefinite integrals using GeoGebra. By emphasizing conceptual understanding, technological fluency, and instructional competence, the module addresses critical challenges associated with adopting digital pedagogies in mathematics education. The findings indicate the need for sustained and structured professional growth supported by access to digital tools and collaborative environments, with important implications for school administrators. For teacher educators, this study offers an evidence-based framework for facilitating professional growth through targeted activities, iterative Evaluations, and reflective practice, which can inform mathematics methods courses and ongoing professional growth programs. For policymakers, the research highlights the need to coordinate curriculum standards, evaluation methods, and professional learning with blended classroom requirements. This alignment seeks to improve student involvement and learning effects while guiding scalable, sustainable STEM professional growth models.

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Research Ethics. Not Applicable

Data Available Statement. The data can be accessed through the official figshare repository at https://figshare.com/s/4d4d883887bdb84fe04d.

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